Limited Information and Entry Decision:  
A source of Entrepreneurial Overconfidence

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This paper seeks to explain the persistence of earning differentials between wage earners and the self-employed. I develop a simple model based on Denrell Jerker's (2003) insight, which points to the importance of having all of the information in decision making. I use a Bayesian setting and concentrate specifically on the entrepreneur’s entry decision. I show how limited information can lead to substantial overestimation of potential profits even if the decision-maker has unbiased prior beliefs, which essentially creates an overconfident entrepreneur. The driving force behind the conclusion is the assumption that potential entrepreneurs are not aware of the fact that information available to them is limited. The resulting bias leads to overconfidence, excessive entry and the persistence of lower returns to entrepreneurship when compared to returns in the wage sector.

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1. Introduction

One of the major stumbling blocks for researchers studying entrepreneurship has been the fact that average returns to business ownership are too low. More specifically, entrepreneurs (on average) earn less than their wage earning counterparts (Hamilton 2000). This finding is true both for initial earnings and for earnings growth differentials. This paper seeks to provide an explanation for these differentials and for their persistence. The realization of this stark earnings difference is quite recent (older studies either did not find a difference or could not estimate self-employed earnings reliably). It is mostly due to the fact that the true profits of entrepreneurs are an elusive variable. One of the common beliefs about business ownership is that founders often underreport their true profits by overstating their business expenses. Some early studies (Brock & Evans, 1986; Rees & Shah, 1986; Borjas & Bornars, 1989; Evans & Leighton, 1989) found no significant differences in earnings between the self-employed and wage earners. In contrast, a more recent and more detailed examination has shown clear differences (Hamilton, 2000).

There are several problems from which previous studies suffered. First, the superstar theory proposed by Rosen (1981) may play a big role here; this theory suggests that wage to profit comparisons of averages may not be an appropriate measure since profits are highly skewed towards few superstars at the right tail of profit distribution. An equally important problem is that data used in all of these papers lacks important information such as the length of business ownership (Hamilton 2000). Finally, net profits (as reported to the IRS) are used as a typical measure of self-employment income. Reporting less income to the IRS leaves a business' owners with more earnings to keep. Hamilton (2000) took several steps towards addressing these concerns. For example, he tracked business ownership over ten years, which allowed him to construct a better earnings approximation for the self employed. This has enabled him to get a better picture of business' profits since choices to sell the venture reflected some of the unreported true business values. Hamilton used the 1984 Survey of Income and Program Participation (SIPP), which tracks 8,771 male school leavers aged 18-65 over a ten year period (leavers implies all males exiting high school with a diploma, or otherwise). In addition to better approximating business' earnings Hamilton examined the business population more closely (by means of quantile regression) rather than a simple average comparison to address any superstar theory concerns. As a result, he found that self-employed earnings were dominated by incomes of wage earners with similar characteristics (such as experience, age, education, etc.). Two years later another study confirmed this result (Moskowitz and Vising-Jorgensen 2002). These findings beg the question: Why would anyone choose to become an entrepreneur?

This paper offers an explanation of this seemingly suboptimal choice. It points to the importance of information in the decision-making process of potential entrepreneurs. Information plays a vital role when evaluating the expected profits of a venture. Any agent
who enters an industry must initially hold beliefs that their expected future profits will outweigh any entry costs. At the same time, these expected profits should compare favorably to the highest valued alternative. After entry, an agent compares her realized profit to her previous expectations, and updates her beliefs. In this paper I model this process (of entry, and consequential beliefs update) in order to gain insight into the entry and exit dynamics of entrepreneurship. In my model, whenever there are some informational limitations the analysis of future profit potential produces an overestimated value of expected profits. As a result there will be a number of agents entering erroneously, only to exit as soon as actual profits are realized and beliefs are updated. This need not be the case for all agents. As long as there are some who rely on erroneous information, overentry and inefficiency will prevail. As a result, there is a continuous overentry driven by new agents with incorrect expectations, which essentially explains why some entrepreneurs are overconfident. The paper proceeds as follows: Section 2 reviews the literature and examines where this theory fits in; Section 3 develops the model; Section 4 discusses potential implications and tractable avenues for empirical examination; and, Section 5 concludes and summarizes the contributions of this paper.

2. Literature Review

This paper offers a mechanism that may be responsible for the overconfident behavior of the entrepreneur. In this model, an entrepreneur behaves in what seems to be an irrational manner because there is a mistake in the information used for making an entry decision. This model neither excludes nor proves any of the previously posed explanations wrong. Other explanations for this entrepreneurial behavior and observed wage differentials include taste-for-variety, risk aversion, and overconfidence. All of the proposed explanations have some legitimacy and are not mutually exclusive. One of the most obvious ways to explain this seemingly irrational behavior is to assume that there is some additional utility that entrepreneurs get from business ownership. Hamilton himself concluded: “self-employment earning differential reflects entrepreneurs’ willingness to sacrifice substantial earnings in exchange for nonpecuniary benefits of owning a business.” (Hamilton, 2000)

This is essentially an example of the taste-for-variety theory (Åstebro and Thompson 2008). This theory initially started as a psychological hypothesis for taste-to-change-jobs where utility was derived from job changes (Ghiselli 1974). It later evolved to taste for being your own boss and taste for multiple tasks (Benz and Frey 2004). Åstebro and Thompson (2008) examine a sample of 830 surveys from the Canadian Innovation Centre and find significant support for the taste-for-variety theory; basically, agents who like being their own boss choose to open their own ventures.

Another, the oldest explanation, dates back to Adam Smith, Schumpeter and Knight defines the role of an entrepreneur as the risk bearer. This was first formally modeled by
Kihlstrom and Laffont (1979). In Kihlstrom and Laffont’s model, agents differ in their risk preferences; consequently, more risk tolerant individuals choose to be entrepreneurs. This model separates individuals into two groups (workers and entrepreneurs) based on risk. Some inefficiencies may arise from the same assumptions and mechanisms which are the driving force behind their conclusions. Since all agents (including entrepreneurs) have different risk preferences, in equilibrium entrepreneurs choose to produce different amounts of goods. This may lead to inefficiency. Another potential source of inefficiency is that wage earners do not bear any risk at all; thus, if there are risk averse (but able) entrepreneurs, they do not get to put themselves to the best use.

Most recent and the closest to my proposed model explanation is that of the overconfident entrepreneur. In this view, the entrepreneur is not able to estimate her chances for success correctly due to the fact that she holds erroneous beliefs about her ability. Analogous to the Winner’s Curse (Thaler 1992), the most overconfident person undertakes the path of business ownership. The agents not pursuing business ownership are those that are able to estimate their chances for success correctly; hence, realizing that it is not worth it, they choose not to pursue business ownership. Overconfidence theory has an entrepreneur evaluating her chances for success, which in this case means assessing her own ability.

Overconfidence has been studied for some time by psychologists, economists and business scholars. Psychologists have long known that people overestimate low probabilities (chances of winning a lottery) and underestimate high probabilities (chances of getting lung cancer for smokers). Studies of overconfidence in psychology goes back to Kahneman and Tversky (1973), with more recent experiments finding that people update their beliefs only mildly whenever presented with new and relevant information, which leads to overconfidence (Brenner et al 1996). Business literature has also found that status quo bias is an impediment to incorporating relevant information into the decision making-process and therefore leads to biases (Samuelson and Zeckhauser 1988). Rabin (1996) wrote a bridge paper to expose economists to many of these psychological findings, including that of overconfidence. Lowe and Ziedonis (2006) found that entrepreneurs continue unsuccessful development efforts much longer when compared to corporations. But their study did not include length of time in the industry as a variable. This might simply imply that corporations are farther along their learning curve. It also seems that there is a simple selection problem associated with Lowe and Ziedonis (2006). Namely, better start-ups are now actually corporations. In general, the results of studies linking entrepreneurs and overconfidence are mixed.

Brennan and Torous (1999) study the behavior of investors and find that investors tend to be overconfident (all investors); this is mainly driven by less than optimal portfolio diversification. Barber and Odean (2001) find men to be more confident than women.
Results on whether overconfidence is socially efficient are mixed as well. Bernardo and Welch (2000) put forward a model in which the overconfidence of some individuals is essential for group survival. Puri and Robinson (2005) find that overconfident individuals choose to work more, which may be socially optimal. Eric Van Den Steen (2004) formalized choice-driven overoptimism in his paper. He models the evolution and persistence of the overconfidence with most overconfident agents pursuing entrepreneurship. The source of overconfidence in all of the above models is not explicitly modeled--some agents are assumed to be overconfident without explaining how they got there. For example, in Van Den Steen’s model, agents have different priors (ex ante) which lead them to have different posterior beliefs (some end up forming overly optimistic expectations).

In my model, agents end up making suboptimal choices based on the same set of prior beliefs, which is a correct set of beliefs (correct in the sense that it holds information about the true distribution of an industry’s profits). Essentially, in my model, all agents know something about the true profit distribution. They all know the variance, but what is left to estimate is the expected earnings on which an entry decision is to be made. In my model, just like in Steen’s, some agents end up forming an overly optimistic expectation of their profit potential. Biased posterior beliefs are a product of the limited information sample (I assume limited information is a product of competition) collected and analyzed by agents. An agent is evaluating her chances of success; her own ability is kept outside of the picture. Instead, the primary factor responsible for the estimation of future profits is an estimate based on a sample of profit observations collected from the industry. In reality, there are estimates of one’s own ability and industry profit potentials for any agent contemplating entry. However, my model concentrates on the assessment of the state of industry’s profits. More specifically, I look at the potential entrant evaluating the overall profitability of a given industry.

As in previous theories, this paper seeks to provide an explanation for the earnings differentials phenomena. In this model, overentry occurs whenever there is some profit information hidden from an entrepreneur who is contemplating entry. This information is hidden as a result of competition in the industry, which weeds out less successful firms. Over time there are some firms that draw low profits from the profit distribution, whenever these profits are too low to remain in the industry these firms exit. Hence, the remaining population of firms will have a higher average profit. Considering the population of existing firms in the estimation of future profits leads to a substantial bias since all the firms that ever had existed must be included in a true population.

Focusing attention on the problem of limited information has most recently been undertaken by Jerker Denrell (2003). He shows how undersampling may lead to systematic bias. The main argument of his paper is that firms and managers learn from each other, but in practice managers tend to pay more attention to the best managerial practices and to the
best managers. This practice implies that the resulting sample is not representative of the whole manager/firm population. My model draws from the same idea. My goal is to explain the behavior of entry; specifically the entrepreneur’s decision process. I model the Bayesian learning processes explicitly. I first show, how even based on the correct prior information, bias arises and persists. I then explore the properties of the bias as well as possible remedies.

3. The Model

To evaluate the role of information in entrepreneurial entry it is useful to first ask whether potential entrants try to gauge their chances of success using some sort of information gathering process. There is already evidence suggesting that an entrepreneur conducts a search prior to opening a business (Cooper, Folta & Woo, 1995). This paper examines the type and length of search among 1176 new entrepreneurs in a survey conducted by the National Federation of Independent Business (NFIB). Cooper et al (1995) found that potential entrepreneurs with no relevant experience conducted an extensive search prior to entry. These researchers also found that a significant part of entrepreneurs’ searches involved surveying other business owners. The second finding of Cooper et al. is that entrepreneurs with extensive experience conducted a less thorough search. A similar search process, along with a resulting entry decision, is what my model has at its core. In order to gauge agents’ profit potential, entrepreneurs need to collect profit information from existing businesses to update their prior beliefs about the profitability of a venture. My model uses a Bayesian framework, where variances serve as weights in an updating process; a larger weight on prior information makes new information less valuable.

What about the finding that asserts experienced entrepreneurs search less? In essence, a more experienced entrepreneur has already collected a significant amount of information, which now serves as her prior distribution in the updating process. Thus, an experienced potential entrepreneur will spend fewer resources in going after additional information. Important evidence found by Cooper et al is that the all entrepreneurs undertake this assessment (search) prior to entry. My model posits that this assessment is a primary criterion in the decision-making process. Cooper et al essentially examined the evidence for the process of assessment prior to entry, which is in line with the mechanism that I propose. Cooper et al. states that the only theory (Lord & Maher 1990) which sets out to explain information search is not consistent with the evidence. Cooper et al (1995) goes on to point out that Lord and Maher’s (1990) theory suggested the complete opposite to the evidence collected by Cooper et al. While it is not a primary goal of this model to produce a new search theory, the results concerning search are well in line with this evidence provided by Cooper et al (1995).
To model this mechanism by which prior beliefs are updated using new information, several assumptions need to be made. First, I assume that all agents are equal in their ability. Hence, the only factors on which the entry decisions are to be made are prior beliefs about the profitability of a venture and any new information (collected by an agent) which can improve upon those beliefs. For simplicity, I assume that there are two distributions involved in the mechanism: distribution of profits (unknown to the observer) and a distribution of signals. Both of them are normal. Agents are able to estimate future profits based on signals. However, the individual profit draw, which an agent gets upon entry, remains unknown. Let the profit distribution take the following form:

\[ p \sim N(\mu, \sigma_p^2) \]  

Agents who choose to enter draw profits from this distribution. Assume that, while the whole distribution cannot be seen by people contemplating entry, the variance of the distribution (\(\sigma_p^2\)) is common knowledge. This information is known to all agents and is essentially an unbiased (correct) prior belief. An entrepreneur who draws profit from this distribution will earn this profit from the first period after entry onwards forever. Incumbent firms communicate their true profits to potential entrants via signals \(s_i\) (whether they want to or not); these signals are stochastically related to agents’ true profits. I assume that firms cannot think strategically when it comes to their signals; they have to report their profits honestly. This way, the incumbents cannot misinform the observer and deter entry. Assume a signal is characterized by:

\[ s_i = p + \varepsilon \]  

with \(\varepsilon \sim N(0, \sigma_e^2)\). Here, I also assume that the precision of the white noise is known to the potential observer. A collection of these signals could be viewed as the new information available to an entrant. These signals are used in a likelihood function to update prior beliefs and to form posterior beliefs. Posterior profit expectations could be estimated, which will enable agents to make an informed entry choice. The driving assumption behind the conclusions is the following: I assume that only operating firms can send these signals to an observer. Firms that have exited (in previous periods) no longer post this information. In reality, this information is probably obtainable but it is costly. I make these costs prohibitive in order to simplify the analysis and to have cleaner results. Immediately, it is clear that agents could differ in two respects: 1) agents know that this inaccessible information (profits of firms that have failed) exists; and, 2) agents are not aware of the fact that some information is missing. For simplicity, call them “Informed” and “Uninformed” agents, respectively. I further assume that entrepreneurs cannot make buying offers to one another, making it impossible for informed agents to take advantage of the uninformed. Since the
distribution of profits is truncated from the left it essentially hides a portion of signals from an observer. Another important assumption is that those firms which perform poorly fail and exit. This supposition enables me to simplify exit decisions to a one dimensional analysis in which only profits and alternative wages matter to entrepreneurs. Entrants end up using a simple decision rule: getting high enough profit means staying while drawing a low profit forces them to immediately exit. Let $\beta$ represent the truncation point separating failed firms from incumbents. An observer collects $n$ signals from a Normal distribution with an unobserved mean and $\sigma^2$ variance.

Applying the standard Bayesian formula (e.g. de Groot, ch. 9), the posterior belief of an uninformed agent is:

$$E(p|s_1, s_2, ..., s_n) \equiv \mu_n = \frac{\mu \sigma^2}{\sigma^2 + n \sigma^2} + \frac{\sigma^2 \sum_{i=1}^n s_i}{\sigma^2 + n \sigma^2}$$

Because the uninformed agent believes she is well informed she just assumes that the observed $n$ signals are drawn from a standard normal distribution. At the same time, an informed agent is well aware of the fact that the distribution is truncated such that $s_i \geq \beta$. She is, however, interested in a true value of $\sum_{i=1}^n s_i$, which would have been observable if the distribution was not truncated. The expected value of the truncated signals is:

$$E[s] = \hat{\beta} + \sigma_e \frac{e^{-1/2(\beta-\hat{\beta})^2/\sigma^2_e}}{\int_{\beta-\hat{\beta}}^{\infty} e^{-1/2t^2/\sigma^2_e} dt}$$

Where we denote agent’s maximum likelihood estimate of profit as $\hat{\beta}$. This estimate is based solely on the information available to an observer. Then, $\hat{\beta}$ is given by the solution to (4), (see, for example Selvin (1976, eqn \(\equiv 2\))), so the expected value of observed signals is defined implicitly by:

$$\hat{\beta} = \frac{1}{n} \sum_{i=1}^n s_i = \sigma_e \frac{e^{-1/2(\beta-\hat{\beta})^2/\sigma^2_e}}{\int_{\beta-\hat{\beta}}^{\infty} e^{-1/2t^2/\sigma^2_e} dt}$$

Let $\bar{s} = \frac{1}{n} \sum_{i=1}^n s_i$, and let $g(\bar{s})$ denote the above solution. The decision maker knows (or is able to derive) (5). She will then use this estimate in a similar fashion (as in (3)) to obtain:

$$E\left(p|s_1, s_2, ..., s_n; s > \beta \right) = \frac{\mu \sigma^2}{\sigma^2 + n \sigma^2} + \frac{\sigma^2 \sum_{i=1}^n s_i}{\sigma^2 + n \sigma^2}$$

An informed agent will use all of the available information (including relevant information about this truncation), and she then incorporates all of this information in her estimate.
Equation (6) provides the correct Bayesian posterior belief about profit potential. This correct belief is accurate in the sense that it is unbiased. Bias is then determined by subtracting (6) from (3):

\[ B_n = \frac{n \sigma_x \sigma_y^2 (\bar{s} - g(\bar{s}))}{\left( \sigma_x^2 + n \sigma_y^2 \right)^{\frac{1}{2}}} \int_{-\infty}^{\bar{p}} \frac{1}{\sigma_x^2 + n \sigma_y^2} e^{-\frac{1}{2} (\cdot)^2} dt \]  

Proposition 1: If the likelihood function is truncated from the left, then the observer who uses Bayesian updating will overestimate his/her expected profits. \( B_n > 0 \)

Note immediately that \( B_n = \frac{n \sigma_x (\bar{s} - g(\bar{s}))}{\int_{-\infty}^{\bar{p}} \frac{1}{\sigma_x^2} e^{-\frac{1}{2}(\cdot)^2} dt} \) with \( n \frac{\sigma_x}{\left( \sigma_x^2 + n \sigma_y^2 \right)^{\frac{1}{2}}} > 0 \), and

\[ \bar{s} - g(\bar{s}) = \frac{\sigma_x}{\int_{-\infty}^{\bar{p}} \frac{1}{\sigma_x^2} e^{-\frac{1}{2}(\cdot)^2} dt} > 0 \text{ for all parameters implying} \ B_n > 0 \]

The above proposition states that whenever a potential entrepreneur is observing an industry and does not take previous existing firms (and their profits) into consideration, she is bound to end up with an overly optimistic estimate for her profit potential.

i) Prior Uncertainty

Bias has several parameters that affect its magnitude. I first consider prior beliefs and how the quality of these beliefs affects the resulting estimate. Prior information is captured by \( \sigma_\mu^2 \), which reflects the quality of prior beliefs. Differentiating bias with respect to \( \sigma_\mu \) is straightforward and yields:

\[ \frac{\partial B_n}{\partial \sigma_\mu} = \frac{n \sigma_x \sigma_\mu}{\left( \sigma_x^2 + \sigma_\mu^2 \right)^{\frac{1}{2}}} \left( 1 + \text{erf} \left( \frac{\bar{p}_n - \bar{p}}{\sqrt{2} \sigma_\mu} \right) \right) \]  

This effect is strictly positive for all values of parameters (i.e. \( \partial B_n / \partial \sigma_\mu > 0 \)). This means that the greater one’s prior uncertainty, the worse the bias. This arises from the simple fact that a reduction in the precision of correct information \( (\sigma_\mu^2) \) is the variance of the true profit distribution) forces an observer to put more weight on the biased information captured by the likelihood function. One can think of this as a situation in which this agent has an initial

\[ ^1 \text{For a more formal discussion of posterior moments arising from truncated likelihood, see Balakrishnan and Ma} (2001). \]
idea about profit potential. The profit potential is denoted by an interval. This agent then goes on and collects more information. But, the wider the interval the more uncertain is the agent about what to make of this information. Thus, she trusts this newly collected (biased) information more as a result.

ii) The number of Signals

Does collecting more signals help? Intuition suggests that a deeper search may return better (less biased) results. As it turns out, collecting more signals does not help ($\partial B_n/\partial n > 0$). Collecting more signals means that an observer is actually more misinformed. This is due to the fact that collecting signals from truncated distributions forces an observer to place more weight on the information carried by these signals. Therefore, increasing the number of signals does not alleviate the bias because all other signals are drawn from this incomplete distribution, which is misleading. Thus, getting larger samples of these signals is not only unhelpful in reducing the degree of misinformation, but it even makes the problem worse.

iii) Noisy Signals

If the collected signals are very noisy, then the correlation between an agent’s profit and the profits of incumbents is weak. Knowing that the noise level is high, an observer will not know what to make of this information and will put less weight on the information extracted from signals; this should reduce resulting bias. However, if truncation is severe (truncation point is to the right of the mean), then it may actually help to have less noise. The overall direction of this parameter change is unclear. To see which restrictions are needed to provide a definitive, direction consider the entire expression.

$$
\frac{\partial B_n}{\partial \sigma_e} = \frac{\hat{\beta} - \beta}{\sigma'_e} \frac{\hat{\beta} - \beta}{\sigma'_e} \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\hat{\sigma}^2}}{\sigma'_\mu} - \frac{2e^{-\frac{(\hat{\beta} - \beta)^2}{2\sigma_e^2}}}{\sqrt{2\pi} \sigma'_\mu} \frac{1}{\sqrt{\hat{\sigma}^2}} (1 + \text{Erf} \left[ \frac{\hat{\beta} - \beta}{\sqrt{2\sigma_e^2}} \right])^2 + \frac{2e^{-\frac{(\hat{\beta} - \beta)^2}{2\sigma_e^2}}}{(n\sigma'_e + \sigma'_e)^2} (1 + \text{Erf} \left[ \frac{\hat{\beta} - \beta}{\sqrt{2\sigma_e^2}} \right])
$$

Severe truncation, while mathematically feasible, is not going to be considered in this model. I assume that $\beta < \hat{\beta}$ from now on. A situation in which $\beta > \hat{\beta}$ would imply an entrepreneur is observing an average firm and that this average firm is failing. This should mean that no one at all would enter. A closer look (see Appendix for Proofs) reveals that
\( \beta < \hat{p} \) is not a sufficient condition for the overall effect to be positive. There is a necessary and sufficient condition (see Appendix) which ensures that the derivative is positive.

\[
\frac{\partial B_n}{\sigma_\varepsilon} > 0 \quad \text{if and only if} \quad \frac{2e^{-(\hat{p}-\hat{p})^2/2\sigma_\varepsilon^2} (\hat{p}-\beta)}{\sqrt{2\pi(1-Erf(\frac{\hat{p}-\beta}{\sqrt{2\sigma_\varepsilon^2}}))}} + \frac{2\sigma_\varepsilon^2}{n\sigma_\varepsilon^2 + \sigma_\varepsilon^2} > \sigma_\varepsilon^2 + (\hat{p} - \beta)^2 \tag{10}
\]

If (10) is satisfied, then as signals get noisier the overall bias will get worse. This comes from the fact that the difference between \( \hat{s} \) and \( g(\hat{s}) \) gets larger as \( \sigma_\varepsilon^2 \) gets larger. At the same time, the above condition (10) identifies a range of parameters for which this derivative is negative (if both inequalities are reversed). In this case an observer will place less weight on the signals (incomplete information, which is misleading) thus improving her posterior beliefs.

iv) Variations in truncation point.

If a truncation point is moved to the left, leading to better information, then it should be obvious that there should be less bias as a result. While it is true for the severe truncation case, it is not always the case for mild truncation (see Appendix). An additional condition is needed to insure that increases in truncation increase the bias; namely,

\[
\frac{\partial B_n}{\beta} < 0 \quad \text{if and only if} \quad \sqrt{2\pi} (\hat{p} - \beta) \left( 1 + Erf \left[ \frac{\hat{p}-\beta}{\sqrt{2\sigma_\varepsilon^2}} \right] \right) > 2\sigma_\varepsilon \ e^{-\frac{(\hat{p}-\hat{p})^2}{2\sigma_\varepsilon^2}} \tag{11}
\]

While these findings examine the bias and its properties, the factors that determine this truncation threshold for a particular industry remain outside of this framework. Nonetheless, these are very important. What determines the profits threshold necessary for a particular business? Why do these thresholds differ, and what does that mean for entrants? These questions are not addressed directly by the Bayesian mechanism of this model, but there are obvious determinants of the success thresholds for any particular business. These thresholds are higher for businesses where high entry fees need to be recouped. Increasing the sunk costs of entry decreases the number of entrants. Thus the number of those entering erroneously decreases as well. They (agents who entered misguided) then exit upon the first profit realization. I take higher entry costs to mean that it takes larger profit estimates to fool agents into entry (because agents would expect to recoup these costs over time). A number of agents will enter, only to realize that their expectations were too high. But many will remain in business since entry costs become sunk costs after an entry decision is made. Not exiting and not recouping costs implies migrating to the left side of the profit distribution. We should observe lower rates of failure for high entry costs (such as buying capacity settings – cars, planes manufacturing) as well as for
expensive red-tape businesses (nuclear energy, natural resource extraction). More importantly, this becomes a self sustaining mechanism; industries with low failure rates will continue to have low failure rates, and industries with high failure rates will sustain high failure rates. In case of the low entry costs industries, agents who drew low profits exit (unlike those in high sunk costs cases). Consequently, to the observer it seems that those remaining agents earn a good profit and this observation, paired with low costs of entry, should motivate a lot of entry and as a result more failure as well. These high failure rates will have a larger portion of industry hidden from some observers. This creates a large number of overly optimistic entrants; they enter the industry and fail, thus creating high failure rates.

Even though my model predicts this self feeding mechanism for any given industry, there are changes in these thresholds for the number of industries (Dunne et al 1989). Clearly, there are other mechanisms responsible for these changes. The case could be made for both inter and intra-industry differences and changes in thresholds. Truncation points could change over time due to some exogenous factors such as foreign competition. If we were to examine an industry which has gone through a severe drop in profits (due to foreign competition) it should have higher standards for firms’ survival. Meaning, only the best would remain which would essentially create a new, higher truncation point. In such cases we should see a change in hazard rates as well. Immediately after a shake-out an observer will see fewer firms earning profits (keep in mind that an observer did not monitor the industry prior to shake-out). Fewer incumbent firms imply smaller samples from which estimation is done. Entering entrepreneurs are less misinformed compared to the same industry populated by more firms. Some examples of these types of industries include shoes, steel, knives, wood mills, fisheries, etc. Another factor that may change profit thresholds is some regulatory change which would make entry tougher or easier. Any of these exogenous shocks which change industry dynamics, profit levels, and fixed costs. These shocks will lead to new hazard rates. All of these should change failure rates by varying $\beta$ in distribution of signals and thus affecting the degree of bias.

These are all static comparisons, and saying what happens over time does not fit in well with the current set up of the model. In order to switch to a dynamic setting, we need an agent who can observe a change in the industry and/or who observes several periods (essentially updating her beliefs more than once). As a result of numerous instances of updating, she should be closer to the correct estimate. This is because she observes as a failure as well as success.
4. Dynamics and Empirical Suggestions

This current setup is limited to a one-shot “update-and-decide model”. Even though agents have prior beliefs, and they update those beliefs upon collecting new information, there is nothing to suggest continuous updating. One natural extension is to see what the value of waiting is. I now proceed to extend the model in this direction. Let us consider the very same updating done over “t” periods. Simply, updated expectations for profit potential in period one becomes a prior belief in period two. This dynamic can be captured by the difference equations below:

\[
\mu_{t+1} = \frac{\mu_t \sigma_e^2}{\sigma_{\mu(t)}^2 n + \sigma_e^2} + \frac{\sigma_{\mu(t)}^2 n\bar{S}}{\sigma_{\mu(t)}^2 n + \sigma_e^2} \tag{12}
\]

and

\[
\sigma_{t+1} = \frac{\sigma_e^2 \sigma_{\mu(t)}^2}{\sigma_{\mu(t)}^2 n + \sigma_e^2 n} \tag{13}
\]

Note that in the above equations “t+1” subscripts denote posterior values and “t” subscripts denote prior values. The likelihood function, which includes the latest sample mean and variance, is free of time subscripts to avoid confusion. Thus, \(\bar{S}\) captures an average value of signals from the newly collected sample in this current period. It is important to note that this sample contains the profits from all firms, including those that drew lower than expected profits and are exiting in the same period. One would expect that \(\bar{S}\) in the early periods of observation would contain incumbent firms as well as new entrants and new exits. As observation continues, prior beliefs become less biased and each successive posterior estimation improves in quality. Potential entrepreneurs can now see exiting firms by virtue of observing the industry dynamics given by (12) for numerous periods, without them having to enter. Observing numerous periods should improve an observer’s understanding of the true profit potential. It should be immediately clear that (13) contains only correct information and therefore will not be a source of bias. Recall that the variance of the prior is known to all agents and serves as a correct belief; henceforth, I drop time subscripts on variances. Without loss of generality let us assume that the initial condition for (12) yields number one (it makes no difference, as long as any finite number is chosen). Thus, solving (12) yields a solution for some arbitrary period \(\tau\):

\[
\mu_\tau = -\frac{\bar{S}\sigma_e^2}{\sigma_e^2 + \sigma_{\mu}^2 n} \left(1 - \left(\frac{\sigma_e^2 + \sigma_{\mu}^2 n}{\sigma_e^2 + \sigma_{\mu}^2 n}\right)^\tau\right) + \left(\frac{\sigma_e^2}{\sigma_e^2 + \sigma_{\mu}^2 n}\right)^{\tau-1} \tag{14}
\]

\[
\sigma_\tau = \frac{\sigma_e^2 \sigma_{\mu}^2}{\sigma_{\mu}^2 n + \sigma_e^2 n} \left(1 - \left(\frac{\sigma_e^2 + \sigma_{\mu}^2 n}{\sigma_e^2 + \sigma_{\mu}^2 n}\right)^\tau\right) + \left(\frac{\sigma_e^2}{\sigma_e^2 + \sigma_{\mu}^2 n}\right)^{\tau-1}
\]
One would expect bias to disappear in this case if observation continues infinitely. As our potential entrepreneur begins her observation in any period, she is initially misinformed. Her first estimate will be severely influenced by the fact that the industry has been around for some time in conjunction with the assumption that previous failures are not taken into consideration. But as she continues to monitor the industry, her estimates improve (since now complete dynamics are visible). Eventually, the updating process outweighs initial misinformation. There is new and complete information. Thus, if an agent begins her observation at any given period “s” and continues to update her beliefs as in (12), she is then updating her \( \mu_s \) which is initially biased by observing \( \bar{s} \). Note, this average \( \bar{s} \) is less biased after numerous periods and represents an average drawn from the complete signal distribution because all of the signals from the beginning of observation are now included. But will this bias be eliminated completely? Taking the limit of equation (14) yields:

\[
\lim_{t \to \infty} \mu_t = \bar{s}.
\]

This is an intuitive result. It states that longer observation of an industry reduces bias. An agent observing the complete lifespan of an industry is not misinformed at all. This is an important point in and of itself. Consider the beginning of any industry: agents that entered have not been misled by years of failure. Agents that have started at time zero have no information to rely on. And, in this case this information is not misleading to early entrants. But one must be careful about the term “early entrant”. The term “early entrant” could mean two different things. First, an early entrant is an agent who has entered an industry in the early stages of the industry’s lifecycle. Second, an early entrant is an agent who decided to wait less before making an entry decision. This model states that the former would be less misinformed by definition while the latter is really misinformed. She can use some observing time to remedy the information driven bias (reduce the bias by observing the industry’s dynamics).

The number of periods for which information is missing matters as well. An agent entering some industry in its infant stage does not have many firms that had previously exited. Take an extreme case where there are few firms in the industry and not one firm has failed in the previous few periods. An observer contemplating entry into this industry is not misinformed at all; in fact, she observes the entire profit distribution. Whereas an industry that has been able to weed out a number of firms over its lifetime potentially misleads an observer, suggesting that incumbent firms’ profits are representative of the whole population. In this case an agent can improve the quality of her expectations by observing several periods completely (failures included).
i) **Number of Signals**

I now turn my attention to other possible changes that may take place over time. I first consider changes in the number of incumbent firms. The number of signals could either change over time due to some exogenous factors or as a result of internal entry and failure dynamics. Exogenously dictated differences are, for example, those industries which have different sizes and thus different number of signals *ex ante*. These industries can be compared to one another. Alternatively, the number of signals could change with time due to some outside factors. Resulting differences in the number of incumbent firms will change hazard rates from one industry to the next based on different $n$. In the case where the number of signals increases due to the internal dynamics of an industry, it has to be driven by larger number of firms drawing profits above the exit threshold, put it more simply, there is a decrease in failure rates. At the minimum, this should imply that the profit distribution’s left tail is being pulled in (same result could be due to the whole distribution stretching up, and pulling both tails in), in which case less distributional mass falls below the exit threshold. We should see less profit variance in this type industry and more firms similar to one another. The whole industry should begin to look more homogeneous, with fewer small firms and fewer large firms in term of profits. This intuition could only be used if the overall profit distribution is unchanged otherwise (i.e. profits for the industry and entry rates will remain fixed). If, however, there is something else going on which effects the shape or the size of a profit distribution (i.e. changes in entry and exit rate or movement within distribution) the above conclusions cannot be reached. For example, in a growing industry an increasing number of signals could be the outcome of expansion. In this case, not much could be predicted about changes in distribution. In the case of a declining industry, changes in the distribution of profits are also an outcome of incumbent firms capturing the market share of exiting firms. So it is difficult to analyze these dynamics since for most industries the life cycle tends to be increasing at first and declining thereafter (Horvath et al 2001, Klepper and Simons 1993). It is then likely that some exogenous factors are responsible for this single-peak evolution process.

A more interesting case to consider is that of industries which are different in size due to market demand. Larger industries (in terms of number of firms) should have more misinformed entrants when compared to smaller industries. This scenario holds true for industries with low or no entry costs and should be especially true for industries populated by small firms. Smaller firms tend to be self funded. Therefore, entry choices are made by an entrepreneur based on search rather than by capital providers like venture capitalists or banks based on their assessment. Differences can arise from the fact that a person contemplating entry collects information from existing firms and updates her beliefs in Bayesian fashion. Thus, the number of signals matters. Consequently, in the case of banks and venture capitalists they (capital providers) have their own information and their own
criteria for funding. A search conducted by an entrepreneur matters less. Banks and venture capitalists have their own assessment formulas and techniques based on a much wider sample. Banks and venture capitalists have been observing an industry for much longer than any potential entrepreneur. Hence, having this larger quantity of signals drawn from truncated distribution will create a more biased posterior belief about potential profitability of a future venture. At first glance, the current evidence (correlating the number of firms to hazard rates) does not support this intuition (comparison was made based on Baldwin et al. 2000 study). Perhaps this suggests that a search is more local and would not be reflected in nationwide statistics about the size of an industry. Maybe more controls would need to be implemented (concentration ratios, industry life cycles, and macroeconomic variables). Obviously, clustering effects and geographical distribution will vary greatly depending on an industry. A look into a local population (city and metro areas) could be a more informational examination of this theory. Study examining search before entry do indeed point to a more local search (Cooper et al. 1995).

\( ii\) \hspace{0.5cm} \textit{Variances} \\

Changes in variances also have interesting effects on posterior beliefs. Altering either one or both the variance of the prior beliefs or the variance of new signals being collected will result in changes to posterior expectations. Over time increases in the prior’s variance means that new information from one period to the next is diminishing in quality (since a posterior in period one becomes a prior in period two). The prior variance is the “correct” information in the sense that it is made up of unbiased beliefs about the true population and of all the information gathered from observing a certain number of time periods completely (an observer must have seen a full population of signals since the initiation of industry observation). These complete observations include profits of all successful and failed firms from the beginning of observation. Increases in this variance means that this correct information has decreased in quality and, as a result, it will be trusted less. An important conclusion is that earlier entrants (in the sense of entering in the early days of an industry) are misinformed less. Thus, there should be more successes among them. This model does not produce mechanisms which would be responsible for this change in variances; these mechanisms are responsible for a lot of industry dynamics and need to be investigated. Nonetheless, even if one was to take these changes as completely exogenous and apply it to this model, hazard rates will change. This can be empirically tested by a close examination of the particular profit distributions.

Literature on the life cycles of an industry has been pointing to the fact that many industries evolve in a single peak fashion, with the number of firms and profits initially going up and then steadily declining over time (Horvath et al 2001, Klepper & Simons 1993). Klepper and Simons (1993) also show early entrants to be more successful when
compared to the later ones. And finally, the literature points to industry concentration decreasing during the period of growth, and increasing in decline period (Horvath et al 2001, Klepper and Simons 1993). This is consistent with my model’s predictions about failure in conjunction with changes in prior variance. As prior information decreases in quality (precision drops), more misinformed agents enter and immediately fail. Incumbents then get the market share of the exiting firms. Clustering and geography may once again prove to be of importance here. As an industry grows (both geographically and in product variety), it is harder to keep track of all of the relevant information out there (since search is found to be local by Cooper et al 1995). This implies the quality of prior beliefs decreases.

Increasing the new signal’s variance is a more helpful method of reducing the bias. Agents collecting and analyzing signals will trust (put less weight) on this “incorrect” (incorrect in the sense that the bias arises from the fact that it is not complete) information consequently reducing the bias. In this case earlier (in terms of industry’s age) entrants will have lower failure rates compared to the late ones because of more unobserved failures over time.

Another interesting story is the interaction of the two variances. For example, if both are increasing but the variance from new signals increases faster (the variance of prior information need not be increasing at all), then it could lead to a situation similar to the observed industry’s life cycle. More specifically a shakeout will occur as the weights of variances switch rankings. Incorrect information enters into a decision making process as a more important component; later, the “correct” information takes over as a more important factor leading to a decrease in entry. The results from one industry to the next will vary because of different fixed entry costs, but as long as there are some entry costs a number of agents will mistakenly enter (hoping to regain fixed costs over time); these agents stay even though the costs cannot be recouped, but they move to the lower tail of the profit distribution. Recall, in the case of no entry costs, agents just stay or exit as soon as profits are revealed. So compared to the no entry costs case, a different fraction of entering agents will immediately exit. For that reason, fixed costs will also have a significant role in determining hazard rates in any particular industry.

5. Conclusion

This model shows how seemingly irrational behavior of new entrants can be explained by a lack of complete information. The overestimation of future earnings potential and subsequent over entry is driven by the assumption that income information containing profit levels of firms who have gone under is not accessible. Whenever an observer contemplating entry is not aware of the limited nature of information, she would be bound to overinflate her estimate of earnings potential. This result arises even though the entrant
has a correct prior belief. This correct information gets distorted during Bayesian updating, involving an analysis of incomplete information. I also find that increasing the number of signals during sampling is likely to make the resulting bias worse. On the other hand, collecting information about the entire profit population (observing for several periods prior to entry) helps to reduce bias. For any agent having more precise information about prior beliefs helps, while having more precise information about current profit distributions is likely to make the bias larger. Dynamic changes in system parameters such as the variances of prior and/or signal distributions, and the number of incumbents produce results which may explain life cycles of an industry as well as the different hazard rates observed. Empirical verification is needed where the hazard rate would serve as a dependent variable. The lifespan of an industry, number of incumbents, and quality of information should serve as independent variables. This paper serves as a pilot investigation into the role of information for the study of entrepreneurship.

Information plays a vital role in any kind of decision-making process and it cannot be neglected. If we assume that entrepreneurship is a resource which needs to be supported, then policymakers need to aid business owners’ to maximize their success. This paper suggests that potential entrepreneurs need to be better informed about their true chances of success. Business school provides an excellent example: most case studies examine successful firms, successful entrepreneurs, and choices that proved to be successful; it may therefore create a false picture of business potential. Some of these “educated” students go on to open their own companies. Beliefs about profit potentials are heavily distorted by these few superstar entrepreneurs (Bill Gates, Warren Buffet and the like); this implies that even estimates of average chances for success may not be the best prior belief. The same logic applies to high school sports, where many students count on becoming professional athletes. Lending institutions chose to fund an entrepreneur or not. A person who does not get a loan attributes her failure to the lender’s lack of faith in her success. Better informed people contemplating start ups at and before the lending stage (regardless of loan approval), will produce more optimal talent allocation and fewer bias driven failures. This does not imply that everyone wishing to open their own companies is capable of doing so. This model can and should be extended in both a theoretical direction and verified empirically. An investigation incorporating Bayesian updating, limited information and entrepreneurial entry decision with the general equilibrium model is a natural extension of this research. Adding strategic interactions, where agent can misinform each other to deter entry, as well as a set up where repeated entrepreneurs are possible and different agents will sell businesses to one another because of different expectations are also useful avenues for further investigation.
Appendix

i) Note that $\frac{\partial B_n}{\partial n}$ is clearly positive since:

$$\frac{\partial B_n}{\partial n} = \frac{\sigma_{\xi} \sigma_{\xi}^2 (3-g(\xi))}{(\sigma_{\xi}^2 + n\sigma_{\mu}^2) \int_0^\infty \frac{e^{-t^2}}{\sigma_{\xi}} \, dt}$$
with all parameters larger than zero and $E(\bar{\xi} - g(\bar{\xi}))$ free of $n$ as well.

ii) In the case of $\frac{\partial B_n}{\partial \sigma_{\xi}}$, there are no clear-cut signs of the derivative; parameters produce both positive and negative results. Nonetheless, some bounds could be established, for example in order for $\frac{\partial B_n}{\partial \sigma_{\xi}} > 0$ there is a necessary and sufficient condition

Proof:

Note that after taking expectations:

$$\frac{\partial B_n}{\partial \sigma_{\xi}} =$$

$$\frac{2e^{-\frac{(\mu-\mu)^2}{2\sigma_{\xi}^2}} - \frac{(\mu-\mu)^2}{2\sigma_{\xi}^2} n(\beta-\mu)\sigma_{\mu}^2}{\pi \sigma_{\xi}^2 (n\sigma_{\mu}^2 + \sigma_{\xi}^2)(1+\text{Erf}(\frac{\mu-\mu}{\sqrt{2\sigma_{\xi}^2}}))} + \frac{2e^{-\frac{(\mu-\mu)^2}{2\sigma_{\xi}^2}}}{\sqrt{\pi} n(\sigma_{\mu}^2 + \sigma_{\xi}^2)(1+\text{Erf}(\frac{\mu-\mu}{\sqrt{2\sigma_{\xi}^2}}))} +$$

$$\frac{2e^{-\frac{(\mu-\mu)^2}{2\sigma_{\xi}^2}} (\mu-\mu)^2}{\pi \sqrt{\pi} (\mu-\mu)^2 n\sigma_{\mu}^2} (A)$$

Now to see how can $\frac{\partial B_n}{\partial \sigma_{\xi}} > 0$ 1 re-arrange and simplify (A) equation to compare first two terms to the second two, the resulting necessary and sufficient condition is then given by:

$$\frac{\partial B_n}{\partial \sigma_{\xi}} > 0 \quad \text{iff} \quad \frac{2e^{-\frac{(\beta-\beta)^2}{2\sigma_{\xi}^2}} (\beta-\beta)}{\sqrt{2\pi}(1-\text{Erf}(\frac{\beta-\beta}{\sqrt{2} \sigma_{\xi}}))} + \frac{2\sigma_{\xi}^2}{n\sigma_{\mu}^2 + \sigma_{\xi}^2} < \sigma_{\xi}^2 + (\hat{\beta} - \beta)^2$$

From which it clearly follows that assuming $\sigma_{\xi}^2 > \frac{2\sigma_{\xi}^4}{n\sigma_{\mu}^2 + \sigma_{\xi}^2}$, $(\hat{\beta} - \beta)^2 > \frac{2e^{-\frac{(\beta-\beta)^2}{2\sigma_{\xi}^2}} (\beta-\beta)}{\sqrt{2\pi}(1-\text{Erf}(\frac{\beta-\beta}{\sqrt{2} \sigma_{\xi}}))} (A)$ are a sufficient and necessary condition for above inequality to hold $\blacksquare$
There is also a range of parameters for which $\frac{\partial B_n}{\partial \sigma_\varepsilon} < 0$. It is clear that reversing the inequality signs produces the two necessary and sufficient conditions for that range of parameters.

iii) $\frac{\partial B_n}{\partial \mu_\varepsilon} > 0$ for all values of parameters, this follows from the fact that differentiating Bias with respect to prior variance yields:

$$
\frac{\partial B_n}{\partial \sigma_\mu} = \frac{2 e^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} n^2 \sqrt{\frac{2}{\pi}} \sigma_\mu \sigma_\varepsilon}{(n\sigma_\mu^2 + \sigma_\varepsilon^2)(1 + \text{erf} \left( \frac{\mu - \beta}{\sqrt{2}\sigma_\varepsilon} \right))}
$$

which can be easily simplified to:

$$
\frac{\partial B_n}{\partial \sigma_\mu} = \frac{2 e^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} n\sigma_\mu}{(n\sigma_\mu^2 + \sigma_\varepsilon^2)^2 (1 + \text{erf} \left( \frac{\mu - \beta}{\sqrt{2}\sigma_\varepsilon} \right))}
$$

It is immediately clear that $\frac{\partial B_n}{\partial \sigma_\mu} > 0$.

iv) $\frac{\partial B_n}{\partial \beta}$ could also take on both positive and negative values depending on parameters since differentiating with respect to the truncation point yields:
where in case of severe truncation (β > μ) it is clear that \( \frac{\partial B_n}{\partial \beta} > 0 \). In the case of milder truncation (β < μ) this result is more difficult to obtain. More specifically an additional condition needs to be imposed. Rearranging (B), while assuming β < μ, it becomes clear that

\[
\frac{\partial B_n}{\partial \beta} > 0 \quad \text{iff} \quad 2\sigma_\varepsilon \ e^{-\frac{(\mu-\beta)^2}{2\sigma^2}} > \sqrt{2\pi} (\bar{\sigma} - \beta) \left(1 + \text{Erf} \left[ \frac{\mu - \beta}{\sqrt{2}\sigma_\varepsilon} \right] \right)
\]

**Dynamics**

Note that while Equation 13 has no impact on bias we need to make sure solution exists and that is finite.

solution to \( \sigma_{t+1} = \frac{\sigma^2_\varepsilon \sigma^2_\mu(t)}{\sigma^2_\mu(t) + n\sigma^2_\varepsilon} \) is \( \sigma_\mu(t) \)

Assuming that initial condition constant is equal to 1. Clearly above solution converges to 0.
References


