Being, Becoming and the Undivided Universe: A Dialogue Between Relational Blockworld and the Implicate Order Concerning the Unification of Relativity and Quantum Theory

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Abstract In this paper two different approaches to unification will be compared, Relational Blockworld (RBW) and Hiley's implicate order. Both approaches are monistic in that they attempt to derive matter and spacetime geometry 'at once' in an interdependent and background independent fashion from something underneath both quantum theory and relativity. Hiley's monism resides in the implicate order via Clifford algebras and is based on process as fundamental while RBW's monism resides in spacetimematter via path integrals over graphs whereby space, time and matter are co-constructed per a global constraint equation. RBW's monism therefore resides in being (relational blockworld) while that of Hiley's resides in becoming (elementary processes). Regarding the derivation of quantum theory and relativity, the promises and pitfalls of both approaches will be elaborated. Finally, special attention will be paid as to how Hiley's process account might avoid the blockworld implications of relativity and the frozen time problem of canonical quantum gravity.

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1 Introduction

> Listening not to me but to the Logos it is wise to agree that all things are one.—Heraclitus

> There remains, then, but one word by which to express the [true] road: Is. And on this road there are many signs that What Is has no beginning and never will be destroyed: it is whole, still, and without end. It neither was nor will be, it simply is—now, altogether, one, continuous . . .—Parmenides

1.1 Modeling Fundamental Reality and Ultimate Explanation: A Schism in Physics

There has been a very long standing debate in Western philosophy and physics regarding the following three pairs of choices about how best to model the universe: (1) the fundamentality of being versus becoming, (2) monism versus atomism and (3) algebra versus geometry broadly construed; more generally, which of the myriad formalisms will be most unifying.

Regarding 1, from very early on Western thinkers have generally assumed that everything can be explained. Perhaps the cosmological argument for the existence of God is the classic example of such thinking. In that argument Leibniz appeals to a version of the principle of sufficient reason (PSR) which states [1] "no fact can be real or existing and no statement true without a sufficient reason for its being so and not otherwise." Leibniz uses the principle to argue that the sufficient reason for the "series of things comprehended in the universe of creatures must exist outside this series of contingencies and is found in a necessary being that we call God" [1]. While physics dispensed with appeals to God at some point, it did not jettison PSR, merely replacing God with fundamental dynamical laws, e.g., as anticipated for a Theory of Everything (TOE), and initial conditions (the big bang or some condition leading to it). In keeping with everyday experience a very early assumption of Western physics—reaching its apotheosis with Newtonian mechanics—is that the fundamental phenomena in need of explanation are *motion* and *change in time*, so explanation will involve dynamical laws most essentially.

In the quest to unify all of physics, it is the combination of PSR plus the dynamical perspective writ large (call it dynamism) that has in great part motivated the particular kind of unification being sought, i.e., the search for a TOE, quantum gravity (QG) and the like. Therefore, almost all attempts to unify relativity and quantum theory opt for becoming (dynamism) as fundamental in some form or another. Such theories may deviate from the norm by employing radical new fundamental dynamical entities (branes, loops, ordered sets, etc.), but the game is always dynamical, broadly construed (vibrating branes, geometrodynamics, sequential growth process, etc).

However, it is also important to note that from fairly early on in Western physics there have also been adynamical explanations that focused on the role of the future in explaining the past as well as the reverse, such as integral (as opposed to differential) calculus and various least action principles of the sort Richard Feynman generalized to produce the path integral approach to quantum mechanics. And of course there are the various adynamical constraints in physics such as conservation laws and the

symmetries underlying them that constrain if not determine the various equations of motion. But nonetheless, dynamism is still the reigning assumption in physics.

Dynamism then encompasses three claims: (A) the world, just as appearances and the experience of time suggest, evolves or changes in time in some objective fashion, (B) the best explanation for A will be some dynamical law that "governs" the evolution of the system in question, and (C) the fundamental entities in a TOE will themselves be dynamical entities evolving in some space however abstract, e.g., Hilbert space. In spite of the presumption of dynamism, those who want fundamental explanation in physics to be dynamical and those who want a world that evolves in time in some objective fashion, face well-known problems concerning: (1) the possible blockworld implications of relativity (both special and general) and (2) canonical QG, the quantization of a generally covariant classical theory leading to "frozen time." As for whether relativity (both special and general) implies a blockworld, there is much debate [2]. Regarding special relativity (SR), many of us have argued [3] that given certain widely held innocuous assumptions and the Minkoswski formulation, special relativity does indeed imply a blockworld. In the words of Geroch [4]:

> There is no dynamics within space-time itself: nothing ever moves therein; nothing happens; nothing changes. In particular, one does not think of particles as moving through space-time, or as following along their world-lines. Rather, particles are just in space-time, once and for all, and the world-line represents, all at once, the complete life history of the particle.

In addition there is the problem of time in canonical general relativity (GR). That is, in a particular Hamiltonian formulation of GR the reparametrization of spacetime is a gauge symmetry. Therefore, all genuinely physical magnitudes are constants of motion, i.e., they don't change over time. In short, change is merely a redundancy of the representation.

Finally, the problem of frozen time in canonical QG (unification of GR and quantum field theory) is that if the canonical variables of the theory to be quantized transform as scalars under time reparametrizations, which is true in practice because they have a simple geometrical meaning, then "the Hamiltonian is (weakly) zero for a generally covariant system" [5]. The result upon canonical quantization is the famous Wheeler-DeWitt equation, void of time evolution. While it is too strong to say a generally covariant theory must have $H = 0$, there is no well-developed theory of quantum gravity that has avoided it to date [6]. It is supremely ironic that the dynamism and unificationism historically driving physics led us directly to blockworld and frozen time.

Two basic reactions to this tension between blockworld and frozen time on the one hand and dynamism on the other are to either embrace the former and show that at least the appearances of dynamism, if not the substance, can be maintained with resources intrinsic to relativity or the particular QG scheme in question [2, 7], or reject the former whether conceptually or formally and attempt to construct a fundamental theory that has something definitively dynamical at bottom. The idea is to somehow make time or change fundamental in some way, as opposed to merely emergent as in the case of string theory or an illusion as in the case of Wheeler-DeWitt. Smolin, for example, suggests a radically "neo-Heraclitean" solution wherein change and becoming are fundamental in that axiomatic dynamical laws, the values of constants that

figure in those laws and configuration space itself evolve in time or meta-time [8]. Though he does not necessarily frame it this way, Smolin is advocating for something like a fundamentally Whiteheadian process conception of reality, a process-based physics where change or flux itself is fundamental. In doing so, Smolin joins Bohm and Hiley who have been advocating such an approach for many decades [9].

However, what isn't clear is if Smolin appreciates what a radical departure a process-conception of reality is from atomism wherein reality has some fundamental dynamical building blocks (atoms, particles, waves, strings, loops, etc.) from which everything else is constructed, determined or realized. This brings us to choice point number 2, atomism versus monism. Despite all the tension that quantum theory has created for atomism as originally conceived, most physicists still assume there is something fundamentally entity-like at bottom, however strange it may be by classical lights. But on the process view, potentia, activity, flux or change itself is fundamental, not entities/things changing in time such as particles or strings. In this monistic physics (what Bohm and Hiley call "undivided wholeness"), all talk of such dynamical entities would emerge from, and be derived from, the more fundamental flux together with, and inseparably from, spacetime in a background independent fashion (the formal question remains of course as to how this move would resolve for example the problem of frozen time). Thus Bohm and Hiley are constructing a monistic model wherein "the whole is prior to its parts, and thus views the cosmos as fundamental, with metaphysical explanation dangling downward from the One" [10]. However, the motivations for a process-based physics are not exclusively physical, but are also driven by the desire to have fundamental concepts of physical time correspond with time and change as experienced such that time as experienced isn't merely a subjective psychological feature of humans with no clear physical correspondence. Following Price [11], the key elements to time as experienced are: *objectively dynamical* (flow or flux-like), *present moment* objectively distinguished, and *objective direction*.

This brings us to choice point number 3, algebra versus geometry broadly construed. There is a dizzying array of formalisms at work in physics. In quantum mechanics alone we have matrix mechanics, Schrödinger dynamics, Clifford algebras, and path integrals, to name a few, and in quantum field theory (QFT) we have canonical quantization, covariant quantization, path integral method, Becchi-Rouet-Stora-Tyupin (BRST) approach, Batalin-Vilkovisky (BV) quantization, and Stochastic quantization [12]. When we get to QG and unification the list is even longer and more diverse [13]. Throughout history there have always been differences of opinion, some pragmatic and some principled, about which formalism(s) best models fundamental physical reality. Indeed, one of the striking things about the state of unification is the heterogeneity of formal approaches and the lack of consensus despite the juggernaut of string theory and its progeny. Hiley for example, likes to say that in his program, geometry (spacetime) is derived from algebra (process), rather than the other way around [14]. Other approaches, such as ours, proceed along something closer to the opposite direction. Hiley enumerates several advantages to using orthogonal Clifford algebras in quantum mechanics: (1) they provide a mathematical hierarchy of nested algebras in which to naturally embed the Dirac, Pauli and Schrödinger particles, (2) the approach is fully algebraic, which allows a more general approach

to quantum phenomena, (3) because it is an algebraic theory, it provides a natural mathematical setting for the Heisenberg 'matrix' mechanics, (4) because it is representation free, it avoids the use of multiple indices on spinors, and (5) it removes the *ad hoc* features of the earlier attempts to extend the Bohm approach to spin and relativity [15]. But, what is interesting from the perspective of foundations of physics is that while there is no necessary connection between a formalism and a particular model or metaphysical interpretation, we see that theorists sometimes pick a formalism based in part on their prior metaphysical biases and background beliefs about the nature of reality, in addition to other physical and formal considerations pertaining to unification such as those Hiley gives above. For example, one of the main reasons Hiley adopts an algebraic approach at bottom is that he thinks algebra can better model process whereas the geometrization of time in relativity leads exactly to blockworld, a conception of reality he rejects as too static. Indeed, at least on the surface it is hard to imagine a cosmology less comforting to a process conception of reality than blockworld or $H = 0$. At any rate, what should now be clear is that each of our three choice points has implications for the others.

## 1.2 Prelude: RBW Versus the Implicate Order

In this paper two different approaches to unification will be compared, the Relational Blockworld (RBW) emphasizes being over becoming formally and conceptually, while the Implicate Order of Hiley emphasizes the converse. RBW has something closer to geometry at bottom (discrete graphical structure) while Hiley has Clifford algebras as fundamental. Each of these programs was originally spawned by two diametrically opposed solutions to foundational issues in non-relativistic quantum mechanics (NRQM) and QFT, rather than starting life as models of QG [16–19]. As we will see, while both are cast in the monistic spirit, Hiley's monism resides in Bohm's implicate order and is based on process while RBW's monism resides in "spacetimematter," whereby space, time and matter are co-constructed per a global constraint equation; RBW's monism therefore resides in being while that of Hiley resides in becoming. Both these programs have proposed new formalisms for quantum physics and are in the process of extrapolating their approaches to unification and quantum gravity [14, 20–23].

The Implicate Order of Hiley extends Bohmian mechanics to the relativistic regime and unites spacetime geometry and material processes, as he doesn't want things happening in a background spacetime but wants to "start from something more primitive from which both geometry and material process unfold together" [24]. That which he considers "more primitive" is elementary process. Hiley calls the fundamental process/potentia the "holomovement" and it has two intertwined aspects, the "implicate order" (characterized algebraically) and all the physics derived from it, such as spacetime geometry, the "explicate (or manifest) order." The holomovement is thus the whole ground form of existence, which contains orders that are both implicate and explicate, wherein the latter expresses aspects of the former. Hiley reduces the Clifford algebra $C_{4,1}$ to $C_{1,3}$ whence he derives the vector space of M4 by mapping the Dirac gamma matrices to the orthonormal vectors spanning $V_{1,3}$ of M4. He then defines Bohm momentum and energy densities in the Dirac equation in analogy with his earlier work with Bohm [25]. From the perspective of the implicate

order, rather than point particles being evolved in time aided by instantaneous updating by the quantum potential or pilot wave, the fundamental evolution is one of processes that give rise to explicate structures ("moments" or "durons") extended in space and time. In short, particles and pilot waves are not fundamental but are at best emergent from the implicate order (see Sect. 4). The irony is not lost on Hiley that the Bohm and Hiley work on interpreting NRQM has done more than perhaps any other interpretation to bolster a particle ontology and a "mechanical" conception of the quantum modeled on an analogy with classical mechanics [26]. Indeed, as we will see in Sect. 2, much of Hiley's later work is trying to get out from under such a pseudo-classical model and emphasize the undivided wholeness instead.

However, in order for Hiley to finish his program, presumably, he will need to accommodate any Lagrangian, not just that of the Dirac equation. For example, he will need to compute cross sections for the various collision experiments of high energy physics. If he proceeds along the lines of "current algebra" [27], as suggested by his approach to date [14, 22, 23], perhaps he could produce a Bohmian explanation for why the commutators between some currents in the Standard Model do not close, producing the so-called Schwinger terms. But, even if he were able to find an algebra of process for the Standard Model that provided Bohm momenta and energies for all the particles, he would still have only "a first approximation to the true theory of sub-atomic particles" [28], since the Standard Model is plagued with twenty-some-odd free parameters. He would be in the same boat as everyone else, needing to account for the free parameters of the Standard Model and include gravity (see Sect. 3). The point is that Hiley would have to join the ranks of theorists who are still looking for a 'super-algebra' whence the Lagrangian unifying the Standard Model and gravity.

As with Hiley's implicate order, our account of quantum physics, which we call the Relational Blockworld (RBW), is based on a form of monism, i.e., the unity of space, time and matter at the most fundamental level. We call this fundamental unity "spacetimematter" and use it to recover dynamical or process-like classical physics only statistically. Thus, we do not attempt to derive geometry from algebra but in a sense, the other way round (see Sect. 2). In order to appreciate how GR "emerges" on our view, it is important to understand that, unlike Hiley's account, our approach is fundamentally adynamical and acausal, again, in contrast also to other fundamental theories attempting to quantize gravity (M-theory, loop quantum gravity, causets, etc.). According to RBW, as we will explain in detail in Sect. 2, quantum physics is the continuous approximation of a more fundamental, discrete graph theory whereby the transition amplitude Z is not viewed as a sum over all paths in configuration space, but is a measure of the symmetry of the difference matrix and source vector of the discrete graphical action for a 4D process (Fig. 1a). We have proposed that the source vector and difference matrix of the discrete action in the path integral be constructed from boundary operators on the graph so as to satisfy an adynamical constraint equation we call the "self-consistency criterion" (SCC), (see Sect. 2 for details). While itself adynamical, the SCC guarantees the graph will produce divergence-free classical dynamics in the appropriate statistical limit (Fig. 2a), and provides an acausal global constraint that results in a self-consistent co-construction of space, time and matter that is *de facto* background independent. Thus, in RBW one has an acausal, adynamical unity of "spacetimematter" at the fundamental level that results statistically in the
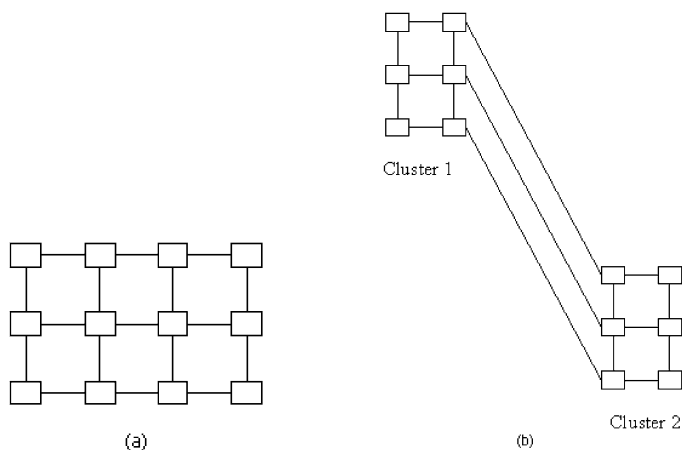
Fig. 1 (**a**) Topological Graph—This spacetimematter graph depicts four sources, i.e., the columns of squares. The graph's actional $\mathbf{K}/2 + \mathbf{J}$, such that $\mathbf{K} \cdot \mathbf{v} \propto \mathbf{J}$, characterizes the graphical topology, which underwrites a partition function Z for spatiotemporal geometries over the graph. (**b**) Geometric Graph—The topological graph of (**a**) is endowed with a particular distribution of spatiotemporal geometric relations, i.e., link lengths as determined by the field values Q on their respective vertices. Clusters 1 & 2 are the result of this geometric process for a particular distribution of field values Q

causal, dynamical "spacetime + matter" of classical physics. This graphical amalgam of spacetimematter is the basis for all quantum phenomena as viewed in a classical context (Fig. 2b), that is, we represent this unity of spacetimematter with 4D graphs constructed per the SCC, and a Wick-rotated Z provides a partition function for the distribution of graphical relations responsible statistically for a particular classical process (Figs. 1 and 2).

Thus, RBW provides a wave-function-epistemic account of quantum mechanics with a time-symmetric explanation of interference via acausal global constraints [17]. Quantum physics is simply providing a distribution function for graphical relations responsible for the experimental equipment and process from initiation to termination. So, while according to some such as Bohmian mechanics, EPR-correlations and the like evidence superluminal information exchange (quantum non-locality), and according to others such correlations represent non-separable quantum states (quantum non-separability), per RBW these phenomena are actually evidence of the deeper graphical unity of spacetimematter responsible for the experimental set up and process, to include outcomes [16, 17]. RBW is therefore integral calculus thinking writ large [16, 20].

As regards the "emergence" or derivation of GR from RBW (see Sect. 3), since we recover classical physics in terms of the "average spacetime geometry" over the graphical unity of spacetimematter, our discrete average/classical result is a modified Regge calculus.[1] Ordinary Regge calculus is a discrete approximation to GR

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[1]Interestingly, in direct correspondence, Hiley noted that he and Bohm had considered Regge calculus, but found it emphasized the 'structure' too much and lost the notion of 'process'. By turning to the notion of an 'algebra', Hiley found he could keep the structure aspect, but emphasize more the process.
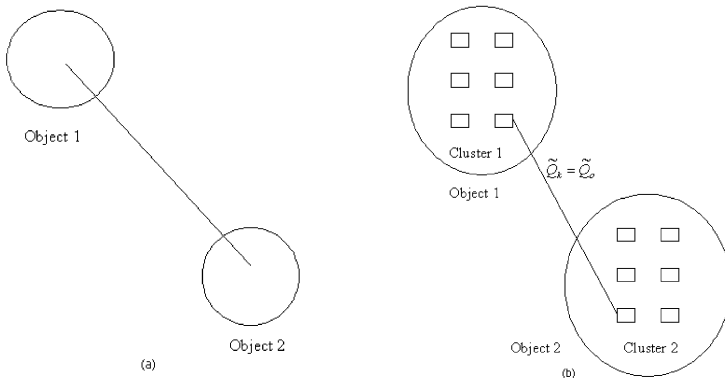
Fig. 2 (**a**) Classical Physics—Classical Objects result when the most probable field values $\mathbf{Q}_0$ yield spatiotemporally localized Clusters 1 & 2 as in Fig. 1b. The lone link in this figure represents the average of the link lengths obtained via the most probable field values $\mathbf{Q}_0$. The most probable values $\mathbf{Q}_0$ are found via $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$, so this is the origin of classical physics. (**b**) Quantum Physics—A particular outcome $\tilde{Q}_0$ of a quantum physics experiment allows one to compute the kth link length of the geometric graph in the context of the classical Objects comprising the experiment, e.g., Source, beam splitters, mirrors, and detectors. The partition function provides the probability of this particular outcome, i.e., $P(\tilde{Q}_k = \tilde{Q}_0) = \frac{Z(\tilde{Q}_k = \tilde{Q}_0)}{Z}$

where the discrete counterpart to Einstein's equations is obtained from the least action principal on a 4D graph [29]. This generates a rule for constructing a discrete approximation to the spacetime manifold of GR using small, contiguous 4D graphical 'tetrahedra' called "simplices." The smaller the legs of the simplices, the better one may approximate a differentiable manifold via contiguous simplices. Our proposed modification of Regge calculus (and, therefore, GR) requires all simplex legs contain non-zero stress-energy contributions (per spacetimematter), so our simplices can be both large and non-contiguous. Consequently, per RBW, GR is seen as a continuous approximation to a modified Regge calculus wherein the simplices can be large and non-contiguous.

Clearly, Hiley's Implicate Order and RBW differ formally (algebraic vs path integral) and conceptually (process-oriented vs adynamical). The monistic character of Hiley's process-oriented approach is housed in the implicate order, i.e., the Clifford algebra. That which we observe (the explicate order) is a projection from the implicate order. Thus, the implicate order accounts for EPR correlations, which appear to require quantum non-locality (as in Bohmian mechanics) and/or non-separability in the explicate order of spacetime. The monistic character of RBW is housed in spacetimematter which underwrites the spacetime + matter classical world of our observations. Thus spacetimematter accounts for EPR correlations, which appear to require quantum non-locality and/or non-separability in the spacetime + matter of our classical perspective [16, 17]. Therefore, both approaches want to explain such observed quantum phenomena from a more fundamental theory underneath quantum theory itself, though these are quite opposing fundamental theories. More specifically, both approaches want to derive GR and quantum theory from something more fundamental in a background independent fashion such that the explanation for quantum entanglement and EPR correlations, rather than creating tensions with spacetime

and relativity, requires neither non-locality nor non-separability in spacetime. Rather, such quantum effects (their phenomenology) are explained at the more fundamental level whether graphical or algebraic. In Sect. 2 we provide a brief overview of Hiley's implicate order (details are already published elsewhere) and a technical overview of RBW. In Sects. 3 and 4 we explore their respective prospects for providing progress in the quest for unification and quantum gravity, and discuss their perspectives on dynamism.

2 Quantum Field Theory: Implicate Order Versus RBW

2.1 Hiley's Implicate Order

Hiley has issued the following challenge [24]:

> Since the advent of general relativity in which matter and geometry code-termine each other, there is a growing realisation that starting from an *a priori* given manifold in which we allow material processes to unfold is, at best, limited. Can we start from something more primitive from which both geometry and material process unfold together? The challenge is to find a formalism that would allow this to happen.

Hiley then refers to Bohm's early attempt [24]:

> David Bohm introduced the notion of a discrete structural process in which he takes as basic, not matter or fields in space-time, but a notion of 'structure process' from which the geometry of space-time and its relationship to matter emerge together providing a way that could underpin general relativity and quantum theory.

While Hiley's view may seem radical to some, he is not alone in appreciating what quantum theory and GR have wrought and what their unification may require [30]:

> General relativity (GR) altered the classical understanding of the concepts of space and time in a way which…is far from being fully understood yet. QM challenged the classical account of matter and causality, to a degree which is still the subject of controversies. After the discovery of GR we are no longer sure of what is spacetime and after the discovery of QM we are no longer sure of what matter is. *The very distinction between space-time and matter is likely to be ill-founded*… I think it is fair to say that today we do not have a consistent picture of the physical world. [italics added]

With regard to QFT, Hiley's own response to his challenge employs "Clifford algebras taken over the reals" to provide "a coherent mathematical setting for the Bohm formalism." In particular, he is concerned with finding the Bohm momentum and energy in a relativistic theory, i.e., the Dirac theory, since a common criticism of Bohm's view is that it cannot be applied in the relativistic domain. Early attempts by Bohm at making his approach relativistically invariant focused on the conserved Dirac current $J^\mu = \langle \bar{\Psi} | \gamma^\mu | \Psi \rangle$ which results from global gauge invariance $\psi \to e^{i\theta} \psi$. Hiley finds

another conserved current associated with the Dirac particle, the energy-momentum density current $2iT^{\mu0} = \psi^\dagger(\partial^\mu\psi) - (\partial^\mu\psi^\dagger)\psi$ which results from invariance under spacetime translations. Hiley argues that this energy-momentum density current is the relativistic counterpart to Bohm energy and momentum for the Schrödinger particle, $E_B = -\partial_t S$ and $\vec{p}_B = \nabla S$. This differs from the standard treatment of the Dirac particle whereby the energy-momentum current is only integrated for global conservation of energy and momentum. In standard field theory, the Dirac current is stressed, since it couples to the gauge field. Hiley's view leads to a curious split of the Dirac particle into a 'Bohm' part and a 'gauge' part. The split is unique to the relativistic regime, as there is no such split for the Schrödinger or Pauli particles. So, what does this relativistic dual nature suggest?

Hiley speculates it is indicative of a composite or extended nature of the Dirac particle. While this idea would apply to baryons, as they are understood as extended and composed of quarks, it would not appear relevant to leptons, which are understood as point-like and fundamental. And what, for example, would we expect for a Bohmian explanation of the twin-slit experiment using Dirac particles? Would the resulting interference pattern be explained by trajectories for the energy-momentum density current in analogy with the Bohmian Schrödinger particle? If so, how would the change in this interference pattern in the Aharanov-Bohm experiment be explained? Since it is the Dirac current that couples to the gauge field and it is the gauge field that is responsible for the Aharanov-Bohm shift in the interference pattern, we would expect the Bohmian trajectories to adhere in some respect to the Dirac current. We suspect that this is indicative of an underlying problem, i.e., trying to understand relativistic quantum phenomena in the context of a particular Lorentz frame, as is done by generating his minimal left ideal with the idempotent $\epsilon_1 = (1 + \gamma^0)/2$. We don't see any problem with his suggested correspondence between his Dirac energy-momentum density current and its non-relativistic, non-spin limit of the Bohm energy and momentum for the Schrödinger particle, i.e., $\rho E_B = T^{00}$ and $\rho P_B = T^{k0}$. However, the fact that it is the energy-momentum density current that makes this correspondence, rather than the Dirac current, suggests to us a breakdown in the Bohmian view (quantum potential defined per a particular Lorentz frame), as would be expected when going to the relativistic regime.

Regardless of whether or not the notion of Bohmian trajectories can be preserved in the relativistic regime, Hiley's implicate order does offer a process-based approach to quantum physics via "a hierarchy of Clifford algebras which fit naturally the physical sequence: Twistors $\rightarrow$ relativistic particle with spin $\rightarrow$ non-relativistic particle with spin $\rightarrow$ non-relativistic particle without spin" [31]. And this approach does unite spacetime geometry and material process via the primitive notion of process algebra. What is unique about the shadow manifolds (explicate order) that are projected from his Clifford algebras is that they lead to an equivalence class of Lorentz observers, rather than a single Minkowski spacetime manifold (M4). Any particular Lorentz frame serves as the base space for a Clifford bundle. Assuming this base space is a flat Riemannian manifold M, Hiley constructs a derivative D from space-like derivatives on M and the generators of his Clifford bundle. Thus defined, D is a connection on M and the momentum operator of quantum mechanics (Schrödinger, Pauli, Dirac equations). He then uses this D to construct a Hamiltonian whence "the two dynamical equations that form the basis of the Bohm approach to quantum mechanics—a

Louville type conservation of probability equation and a quantum Hamilton-Jacobi equation" [32]. While it may seem like a weakness that he produces shadow manifolds rather than M4, we see this as a potential advantage in dealing with the problems of blockworld and "frozen time," as we will discuss in Sect. 4. For now, we simply point out the obvious challenge, i.e., he must find a connection with curvature for the tangent space bundle to the base space manifold so as to recover GR. He speculates this might be done by analyzing phase information in the exchange of light signals, since "the Moyal algebra for relating phase information can be obtained from a deformed Poisson algebra, which is obtained via the hidden Heisenberg algebra" [33]. As he has not begun this project, we can offer only limited speculation on such an attempt in Sect. 3.

Our more general concern is about Hiley's motivation for wanting to obtain a complete relativistic version of the Bohm model for the Dirac particle, given that he clearly rejects the fundamentality of particles and pilot (guide) waves, they are emergent at best. Consider the following passages from Hiley:

> We strive to find the elementary objects, the quarks, the strings, the loops and the M-branes from which we try to reconstruct the world. Surely we are starting from the wrong premise. Parker-Rhodes (1981) must be right, so too is Lou Kauffman (1982)! We should start with the whole and then make distinctions. Within these distinctions we can make finer distinctions and so on [34].

> In this paper we want to draw specific attention to a sixth advantage, namely, that it allows us to apply Clifford algebras to the Bohm approach outlined in Bohm and Hiley. In fact it provides, for the first time, an elegant, unified approach to the Bohm model of the Schrödinger, Pauli, and Dirac particles, in which we no longer have to appeal to any analogy to classical mechanics to motivate the approach as was done by Bohm in his original paper [35].

When Hiley speaks of analogies to classical mechanics, not only is he jettisoning point particles as fundamental but also the wave function and apparently the guide wave:

> In our approach, the information normally encoded in the wave function is already contained within the algebra itself, namely, in the elements of its minimal left ideals [36].

> Thus we see that at no stage is it necessary to appeal to classical mechanics and therefore there is no need to identify the classical action with the phase to motivate the so-called 'guidance' equation $\vec{p} = \nabla S$ as was done in Bohm's original work [37].

> Then it is not difficult to show that this again reduces, in the non-relativistic limit, to the Bohm momentum found in the Pauli case and reduces further, if the spin is suppressed, to the well-known Schrödinger expression $P_B = \nabla S$. This condition is sometimes known as the guidance condition, but here we have no 'waves', only process, so this phrase is inappropriate in this context [38].

> Thus by choosing $\alpha = \frac{1}{2}$ we see that our ρP_j is simply the momentum density. Furthermore it also means that $\vec{P} = \vec{p_B}$, the Bohm momentum. Because

this can be written in the form $\vec{p}_B = \nabla S$. Some authors call this the 'guidance' condition, but here it is simply a bilinear invariant and any notion of 'guidance' is meaningless [39].

It seems to us that there has always been a tension in Bohm and Hiley's "undivided wholeness" and the pseudo-classical Bohmian mechanics conceived as a modal interpretation of NRQM with particles communicating instantaneously with one another, especially in a relativistic setting. Why spend so much energy trying to recover a relativistic Bohmian version of the Dirac particle complete with particle trajectories when such particles and the guidance wave are at best emergent, and the wave function is merely epistemic? In the earlier work it was thought that the Dirac current would provide a means of calculating particle trajectories [40]. In Hiley and Callaghan's recent work they show that the Dirac current is in fact different from the Bohm energy-momentum current, leaving them with two different sets of trajectories [41]; again, all of which raising the question whether Bohmian trajectories can be recovered in the relativistic case after all. But even if such trajectories can be recovered, what's the point of trying to establish that the Bohmian model is relativistically invariant when Hiley rejects the fundamentality of, if not realism about, that very model? If it's the monism *a la* process that matters most to Hiley, then recovering ordinary quantum mechanics or QFT from the algebraic base is sufficient, nothing is added by recovering a relativistically Bohmian mechanics as the latter is just a competing interpretation of quantum mechanics, one that only makes sense to pursue if you take seriously point particles and pilot waves, which apparently Hiley does not. Furthermore, it isn't enough to render Bohmian mechanics Lorentz invariant, it must also be explained how the non-locality in that model can be squared with the relativity of simultaneity. Presumably this problem would get solved by Hiley at the level of the implicate order as a kind of conspiracy theory, but again, then why bother with recovering Bohmian trajectories and the like? In the next section we will see that these problems don't arise for RBW because that model makes a much cleaner break from the ontology of particles and wave functions even at the level of ordinary quantum mechanics in spacetime.

2.2 RBW and Spacetimematter

We believe the real issue is the fact that QFT involves the quantization of a classical field [42] when one would rather expect QFT to originate independently of classical field theory, the former typically understood as fundamental to the latter. Herein we propose a new, fundamental origin for QFT. Specifically, we follow the possibility articulated by Wallace [43] that, "QFTs as a whole are to be regarded only as approximate descriptions of some as-yet-unknown deeper theory," which he calls "theory X," and we propose a new discrete path integral formalism over graphs for "theory X" underlying QFT. Accordingly, sources **J**, space and time are self-consistently co-constructed per a graphical self-consistency criterion (SCC) based on the boundary of a boundary principle [44] on the graph $(\partial_1 \cdot \partial_2 = 0)$.[2] We call this amalgam "space-timematter." The SCC constrains the difference matrix and source vector in Z, which

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[2]In a graphical representation of QFT, part of **J** represents field disturbances emanating from a source location (Source) and the other part represents field disturbances incident on a source location (sink).

then provides the probability for finding a particular source-to-source relationship in a quantum experiment, i.e., experiments which probe individual source-to-source relations (modeled by individual graphical links) as evidenced by discrete outcomes, such as detector clicks. Since, in QFT, all elements of an experiment, e.g., beam splitters, mirrors, and detectors, are represented by interacting sources, we confine ourselves to the discussion of such controlled circumstances where the empirical results evidence individual graphical links.[3] In this approach, the SCC ensures the source vector is divergence-free and resides in the row space of the difference matrix, so the difference matrix will necessarily have a nontrivial eigenvector with eigenvalue zero, a formal characterization of gauge invariance. Thus, our proposed approach to theory X provides an underlying origin for QFT, accounts naturally for gauge invariance, i.e., via a graphical self-consistency criterion, and excludes factors of infinity associated with gauge groups of infinite volume, since the transition amplitude Z is restricted to the row space of the difference matrix and source vector.

While the formalism we propose for theory X is only suggestive, the computations are daunting, as will be evident when we present the rather involved graphical analysis underlying the Gaussian two-source amplitude which, by contrast, is a trivial problem in its QFT continuum approximation. However, this approach is not intended to replace or augment QFT computations. Rather, our proposed theory X is fundamental to QFT and constitutes a new program for physics, much as quantum physics relates to classical physics. Therefore, the motivation for our theory X is, at this point, conceptual and while there are many conceptual arguments to be made for our approach [16, 17], we restrict ourselves here to the origins of gauge invariance and QFT.

2.2.1 The Discrete Path Integral Formalism

We understand the reader may not be familiar with the path integral formalism, as Healey puts it [45], "While many contemporary physics texts present the path-integral quantization of gauge field theories, and the mathematics of this technique have been intensively studied, I know of no sustained critical discussions of its conceptual foundations." Therefore, we begin with an overview and interpretation of the path integral formalism, showing explicitly how we intend to use "its conceptual foundations." We employ the discrete path integral formalism because it embodies a 4Dism that allows us to model spacetimematter. For example, the path integral approach is based on the fact that [46] "the [S]ource will emit and the detector receive," i.e., the path integral formalism deals with Sources and sinks as a unity while invoking a description of the experimental process from initiation to termination. By assuming the discrete path integral is fundamental to the (conventional) continuum path integral, we have a graphical basis for the co-construction of time, space and quantum sources via a self-consistency criterion (SCC). We will then show how the graphical amalgam of spacetimematter underlies QFT.

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[3]Hereafter, all reference to "experiments" will be to "quantum experiments."

2.2.2 Path Integral in Quantum Physics

In the conventional path integral formalism as used by Zee [47] for non-relativistic quantum mechanics (NRQM) one starts with the amplitude for the propagation from the initial point in configuration space q_I to the final point in configuration space q_F in time T via the unitary operator e^{-iHT}, i.e., $\langle q_F | e^{-iHT} | q_I \rangle$. Breaking the time T into N pieces δt and inserting the identity between each pair of operators $e^{-iH\delta t}$ via the complete set $\int dq |q\rangle\langle q| = 1$ we have

$$\langle q_F | e^{-iHT} | q_I \rangle = \left[\prod_{j=1}^{N-1} \int dq_j \right] \langle q_F | e^{-iH\delta t} | q_{N-1} \rangle \langle q_{N-1} | e^{-iH\delta t} | q_{N-2} \rangle$$

$$\cdots \langle q_2 | e^{-iH\delta t} | q_1 \rangle \langle q_1 | e^{-iH\delta t} | q_I \rangle.$$

With $H = \hat{p}^2/2m + V(\hat{q})$ and $\delta t \to 0$ one can then show that the amplitude is given by

$$\langle q_F | e^{-iHT} | q_I \rangle = \int Dq(t) \exp\left[i \int_0^T dt L(\dot{q}, q) \right], \tag{1}$$

where $L(\dot{q}, q) = m\dot{q}^2/2 - V(q)$. If q is the spatial coordinate on a detector transverse to the line joining Source and detector, then $\prod_{j=1}^{N-1}$ can be thought of as $N-1$ "intermediate" detector surfaces interposed between the Source and the final (real) detector, and $\int dq_j$ can be thought of all possible detection sites on the jth intermediate detector surface. In the continuum limit, these become $\int Dq(t)$ which is therefore viewed as a "sum over all possible paths" from the Source to a particular point on the (real) detector, thus the term "path integral formalism" for conventional NRQM is often understood as a sum over "all paths through space."

To obtain the path integral approach to QFT one associates q with the oscillator displacement at a *particular point* in space ($V(q) = kq^2/2$). In QFT, one takes the limit $\delta x \to 0$ so that space is filled with oscillators and the resulting spatial continuity is accounted for mathematically via $q_i(t) \to q(t, x)$, which is denoted $\phi(t, x)$ and called a "field." The QFT transition amplitude Z then looks like

$$Z = \int D\phi \exp\left[i \int d^4x L(\dot{\phi}, \phi) \right] \tag{2}$$

where $L(\dot{\phi}, \phi) = (d\phi)^2/2 - V(\phi)$. Impulses J are located in the field to account for particle creation and annihilation; these J are called "sources" in QFT and we have $L(\dot{\phi}, \phi) = (d\phi)^2/2 - V(\phi) + J(t, x)\phi(t, x)$, which can be rewritten as $L(\dot{\phi}, \phi) = \phi D\phi/2 + J(t, x)\phi(t, x)$, where D is a differential operator. In its discrete form (typically, but not necessarily, a hypercubic spacetime lattice), $D \to \mathbf{K}$ (a difference matrix), $J(t, x) \to \mathbf{J}$ (each component of which is associated with a point on the spacetime lattice) and $\phi \to \mathbf{Q}$ (each component of which is associated with a point on the spacetime lattice). Again, part of $\mathbf{J}$ represents field disturbances emanating from a source location (Source) and the other part represents field disturbances incident on a source location (sink) in the conventional view of path integral

QFT and, in particle physics, these field disturbances are the particles. We will keep the partition of **J** into Sources and sinks in our theory X, but there will be no vacuum lattice structure between the discrete set of sources. The discrete counterpart to (2) is then [48]

$$Z = \int \cdots \int dQ_1 \cdots dQ_N \exp\left[\frac{i}{2}\mathbf{Q} \cdot \mathbf{K} \cdot \mathbf{Q} + i\mathbf{J} \cdot \mathbf{Q}\right]. \tag{3}$$

In conventional quantum physics, NRQM is understood as $(0+1)$-dimensional QFT.

2.2.3 Our Interpretation of the Path Integral in Quantum Physics

We agree that NRQM is to be understood as $(0 + 1)$-dimensional QFT, but point out this is at conceptual odds with our derivation of (1) when $\int Dq(t)$ represented a sum over all paths in space, i.e., when q was understood as a location in space (specifically, a location along a detector surface). If NRQM is $(0+1)$-dimensional QFT, then q is a field displacement at a single location in space. In that case, $\int Dq(t)$ must represent a sum over all field values at a particular point on the detector, not a sum over all paths through space from the Source to a particular point on the detector (sink). So, how *do* we relate a point on the detector (sink) to the Source?

In answering this question, we now explain a formal difference between conventional path integral NRQM and our proposed approach: our links only connect and construct discrete sources **J**, there are no source-to-spacetime links (there is no vacuum lattice structure, only spacetimematter). Instead of $\delta x \to 0$, as in QFT, we assume δx is measureable for (such) NRQM phenomenon. More specifically, we propose starting with (3) whence (roughly) NRQM obtains in the limit $\delta t \to 0$, as in deriving (1), and QFT obtains in the additional limit $\delta x \to 0$, as in deriving (2). The QFT limit is well understood as it is the basis for lattice gauge theory and regularization techniques, so one might argue that we are simply *clarifying* the NRQM limit where the path integral formalism is not widely employed. However, again, we are proposing a discrete starting point for theory X, as in (3). Of course, that discrete spacetime is fundamental while "the usual continuum theory is very likely only an approximation" [49] is not new.

2.2.4 Discrete Path Integral is Fundamental

The version of theory X we propose is a discrete path integral over graphs, so (3) is not a discrete approximation of (1) & (2), but rather *(1) & (2) are continuous approximations of (3)*. In the arena of quantum gravity it is not unusual to find discrete theories [50] that are in some way underneath spacetime theory and theories of "matter" such as QFT, e.g., causal dynamical triangulations [51], quantum graphity [52, 53] and causets [54]. While these approaches are interesting and promising, the approach taken here for theory X will look more like Regge calculus quantum gravity (see Bahr & Dittrich [55] and references therein for recent work along these lines) modified to contain no vacuum lattice structure.

Placing a discrete path integral at bottom introduces conceptual and analytical deviations from the conventional, continuum path integral approach. Conceptually, (1)

of NRQM represents a sum over all field values at a particular point on the detector, while (3) of theory X is a mathematical machine that measures the "symmetry" (strength of stationary points) contained in the core of the discrete action

$$\frac{1}{2}\mathbf{K} + \mathbf{J}. \tag{4}$$

This core or *actional* yields the discrete action after operating on a particular vector **Q** (field). The actional represents a *fundamental/topological, 4D description of the experiment* and Z is a measure of its symmetry.[4] For this reason we prefer to call Z the symmetry amplitude of the 4D experimental configuration. Analytically, because we are *starting* with a discrete formalism, we are in position to mathematically explicate trans-temporal identity, whereas this process is unarticulated elsewhere in physics. As we will now see, this leads to our proposed self-consistency criterion (SCC) underlying Z.

2.2.5 Self-consistency Criterion

Our use of a self-consistency criterion is not without precedent, as we already have an ideal example in Einstein's equations of GR. Momentum, force and energy all depend on spatiotemporal measurements (tacit or explicit), so the stress-energy tensor cannot be constructed without tacit or explicit knowledge of the spacetime metric (technically, the stress-energy tensor can be written as the functional derivative of the matter-energy Lagrangian with respect to the metric). But, if one wants a "dynamic spacetime" in the parlance of GR, the spacetime metric must depend on the matter-energy distribution in spacetime. GR solves this dilemma by demanding the stress-energy tensor be "consistent" with the spacetime metric per Einstein's equations. For example, concerning the stress-energy tensor, Hamber and Williams write [56], "In general its covariant divergence is not zero, but consistency of the Einstein field equations demands $\nabla^{\alpha} T_{\alpha\beta} = 0$." This self-consistency hinges on divergence-free sources, which finds a mathematical underpinning in $\partial\partial = 0$. So, Einstein's equations of GR are a mathematical articulation of the boundary of a boundary principle at the classical level, i.e., they constitute a self-consistency criterion at the classical level, as are quantum and classical electromagnetism [57, 58]. We will provide an explanation for this fact later, but essentially the graphical SCC of our theory X gives rise to continuum counterparts in QFT and classical field theory.

In order to illustrate the discrete mathematical co-construction of space, time and sources **J**, we will use graph theory *a la* Wise [58] and find that $\partial_1 \cdot \partial_1^T$, where ∂_1 is a boundary operator in the spacetime chain complex of our graph satisfying $\partial_1 \cdot \partial_2 = 0$, has precisely the same form as the difference matrix in the discrete action for coupled harmonic oscillators. Therefore, we are led to speculate that $\mathbf{K} \propto \partial_1 \cdot \partial_1^T$. Defining the source vector **J** relationally via $\mathbf{J} \propto \partial_1 \cdot \mathbf{e}$ then gives tautologically per $\partial_1 \cdot \partial_2 = 0$ both a divergence-free **J** and $\mathbf{K} \cdot \mathbf{v} \propto \mathbf{J}$, where **e** is the vector of links and **v** is the vector of vertices. $\mathbf{K} \cdot \mathbf{v} \propto \mathbf{J}$ is our SCC following from $\partial_1 \cdot \partial_2 = 0$, and it defines

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[4] In its Euclidean form, which is the form we will use, Z is a partition function.

what is meant by a self-consistent co-construction of space, time and divergence-free sources **J**, thereby constraining **K** and **J** in Z. Thus, our SCC provides a basis for the discrete action and supports our view that (3) is fundamental to (1) & (2), rather than the converse. Conceptually, that is the basis of our discrete, graphical path integral approach to theory X. We now provide the details.

2.2.6 The General Approach

Again, in theory X, the symmetry amplitude Z contains a discrete action constructed per a self-consistency criterion (SCC) for space, time and divergence-free sources **J**. As introduced above and argued later below, we will codify the SCC using **K** and **J**; these elements are germane to the transition amplitude Z in the Central Identity of Quantum Field Theory [59],

$$Z = \int D\phi \exp\left[-\frac{1}{2}\phi \cdot \mathbf{K} \cdot \phi - V(\phi) + \mathbf{J} \cdot \phi\right]$$
$$= \exp\left[-V\left(\frac{\delta}{\delta J}\right)\right] \exp\left[\frac{1}{2}\mathbf{J} \cdot \mathbf{K}^{-1} \cdot \mathbf{J}\right]. \tag{5}$$

While the field is a mere integration variable used to produce Z, it must reappear at the level of classical field theory. To see how the field makes it appearance per theory X, consider (5) for the simple Gaussian theory ($V(\phi) = 0$). On a graph with N vertices, (5) is

$$Z = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} dQ_1 \cdots dQ_N \exp\left[-\frac{1}{2}\mathbf{Q} \cdot \mathbf{K} \cdot \mathbf{Q} + \mathbf{J} \cdot \mathbf{Q}\right] \tag{6}$$

with a solution of

$$Z = \left(\frac{(2\pi)^N}{\det \mathbf{K}}\right)^{1/2} \exp\left[\frac{1}{2}\mathbf{J} \cdot \mathbf{K}^{-1} \cdot \mathbf{J}\right]. \tag{7}$$

It is easiest to work in an eigenbasis of **K** and (as will argue later) we restrict the path integral to the row space of **K**, this gives

$$Z = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d\tilde{Q}_1 \cdots d\tilde{Q}_{N-1} \exp\left[\sum_{j=1}^{N-1}\left(-\frac{1}{2}\tilde{Q}_j^2 a_j + \tilde{J}_j \tilde{Q}_j\right)\right] \tag{8}$$

where $\tilde{Q}_j$ are the coordinates associated with the eigenbasis of **K** and $\tilde{Q}_N$ is associated with eigenvalue zero, a_j is the eigenvalue of **K** corresponding to $\tilde{Q}_j$, and $\tilde{J}_j$ are the components of **J** in the eigenbasis of **K**. The solution of (8) is

$$Z = \left(\frac{(2\pi)^{N-1}}{\prod_{j=1}^{N-1} a_j}\right)^{1/2} \prod_{j=1}^{N-1} \exp\left(\frac{\tilde{J}_j^2}{2a_j}\right). \tag{9}$$

On our view, the experiment is described fundamentally by **K** and **J** on our topological graph. Again, per (9), there is no field $\tilde{Q}$ appearing in Z at this level, i.e., $\tilde{Q}$ is

only an integration variable. $\tilde{Q}$ makes its first appearance as something more than an integration variable when we produce probabilities from Z. That is, since we are working with a Euclidean path integral, Z is a partition function and the probability of measuring $\tilde{Q}_k = \tilde{Q}_0$ is found by computing the fraction of Z which contains $\tilde{Q}_0$ at the kth vertex [60]. We have

$$P(\tilde{Q}_k = \tilde{Q}_0) = \frac{Z(\tilde{Q}_k = \tilde{Q}_0)}{Z} = \sqrt{\frac{a_k}{2\pi}} \exp\left(-\frac{1}{2}\tilde{Q}_0^2 a_k + \tilde{J}_k \tilde{Q}_0 - \frac{\tilde{J}_k^2}{2a_k} \right) \qquad (10)$$

as the part of theory X approximated in the continuum by QFT. The most probable value of $\tilde{Q}_0$ at the kth vertex is then given by

$$\delta P(\tilde{Q}_k = \tilde{Q}_0) = 0 \quad \Longrightarrow \quad \delta\left(-\frac{1}{2}\tilde{Q}_0^2 a_k + \tilde{J}_k \tilde{Q}_0 - \frac{\tilde{J}_k^2}{2a_k} \right) = 0 \quad \Longrightarrow \quad a_k \tilde{Q}_0 = \tilde{J}_k.$$
$$(11)$$

That is, $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ is the part of theory X that obtains statistically and is approximated in the continuum by classical field theory. We note that the manner by which $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ follows from $P(\tilde{Q}_k = \tilde{Q}_0) = Z(\tilde{Q}_k = \tilde{Q}_0)/Z$ parallels the manner by which classical field theory follows from QFT via the stationary phase method [61]. Thus, one may obtain classical field theory by the continuum limit of $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ in theory X (theory X $\rightarrow$ classical field theory), or by first obtaining QFT via the continuum limit of $P(\tilde{Q}_k = \tilde{Q}_0) = Z(\tilde{Q}_k = \tilde{Q}_0)/Z$ in theory X and then by using the stationary phase method on QFT (theory X $\rightarrow$ QFT $\rightarrow$ classical field theory). In either case, QFT is not quantized classical field theory in our approach. In summary:

1. Z is a partition function for an experiment described topologically by $\mathbf{K}/2 + \mathbf{J}$ (Fig. 1a).
2. $P(\tilde{Q}_k = \tilde{Q}_0) = Z(\tilde{Q}_k = \tilde{Q}_0)/Z$ gives us the probability for a particular geometric outcome in that experiment (Figs. 1b and 2b).
3. $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ gives us the most probable values of the experimental outcomes which are then averaged to produce the geometry for the experimental procedure at the classical level (Fig. 2a).
4. $P(\tilde{Q}_k = \tilde{Q}_0) = Z(\tilde{Q}_k = \tilde{Q}_0)/Z$ and $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ are the parts of theory X approximated in the continuum by QFT and classical field theory, respectively.

2.2.7 The Two-Source Euclidean Symmetry Amplitude/Partition Function

Typically, one identifies fundamentally interesting physics with symmetries of the action in the Central Identity of Quantum Field Theory, but we have theory X fundamental to QFT, so our method of choosing fundamentally interesting physics must reside in the topological graph of theory X. Thus, we seek a constraint of $\mathbf{K}$ and $\mathbf{J}$ in our graphical symmetry amplitude Z and this will be in the form of a self-consistency criterion (SCC). In order to motivate our general method, we will first consider a simple graph with six vertices, seven links and two plaquettes for our $(1+1)$-dimensional spacetime model (Fig. 3). Our goal with this simple model is to seek relevant structure that might be used to infer an SCC. We begin by constructing the boundary operators over our graph.

Fig. 3 Graph with six vertices, seven links e_i and two plaquettes p_i
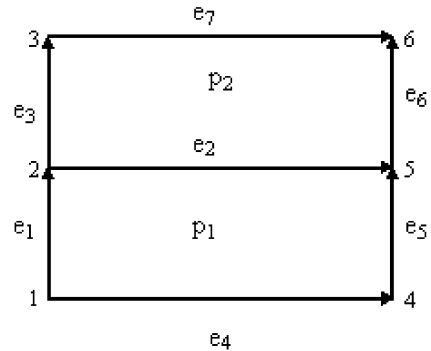
The boundary of p_1 is $e_4 + e_5 - e_2 - e_1$, which also provides an orientation. The boundary of e_1 is $v_2 - v_1$, which likewise provides an orientation. Using these conventions for the orientations of links and plaquettes we have the following boundary operator for $C_2 \to C_1$, i.e., space of plaquettes mapped to space of links in the spacetime chain complex:

$$\partial_2 = \begin{bmatrix} -1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}. \tag{12}$$

Notice the first column is simply the links for the boundary of p_1 and the second column is simply the links for the boundary of p_2. We have the following boundary operator for $C_1 \to C_0$, i.e., space of links mapped to space of vertices in the spacetime chain complex:

$$\partial_1 = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{13}$$

which completes the spacetime chain complex, $C_0 \leftarrow C_1 \leftarrow C_2$. Notice the columns are simply the vertices for the boundaries of the edges. These boundary operators satisfy $\partial_1 \cdot \partial_2 = 0$, i.e., the boundary of a boundary principle.

The potential for coupled oscillators can be written

$$V(q_1, q_2) = \sum_{a,b} \frac{1}{2} k_{ab} q_a q_b = \frac{1}{2} k q_1^2 + \frac{1}{2} k q_2^2 + k_{12} q_1 q_2 \tag{14}$$

where $k_{11} = k_{22} = k > 0$ and $k_{12} = k_{21} < 0$ per the classical analogue (Fig. 4) with $k = k_1 + k_3 = k_2 + k_3$ and $k_{12} = -k_3$ to recover the form in (14). The Lagrangian is

Fig. 4 Coupled harmonic oscillators
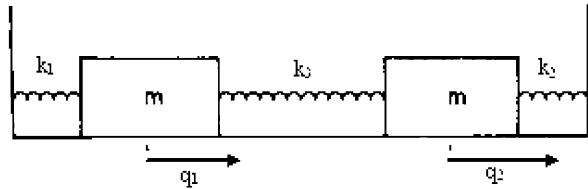
then

$$L = \frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 - \frac{1}{2}kq_1^2 - \frac{1}{2}kq_2^2 - k_{12}q_1q_2 \tag{15}$$

so our NRQM Euclidean symmetry amplitude is

$$Z = \int Dq(t)\exp\left[-\int_0^T dt\left(\frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 + V(q_1,q_2) - J_1q_1 - J_2q_2\right)\right] \tag{16}$$

after Wick rotation. This gives

$$\mathbf{K} = \begin{bmatrix} \left(\frac{m}{\Delta t}+k\Delta t\right) & -\frac{m}{\Delta t} & 0 & k_{12}\Delta t & 0 & 0 \\ -\frac{m}{\Delta t} & \left(\frac{2m}{\Delta t}+k\Delta t\right) & -\frac{m}{\Delta t} & 0 & k_{12}\Delta t & 0 \\ 0 & -\frac{m}{\Delta t} & \left(\frac{m}{\Delta t}+k\Delta t\right) & 0 & 0 & k_{12}\Delta t \\ k_{12}\Delta t & 0 & 0 & \left(\frac{m}{\Delta t}+k\Delta t\right) & -\frac{m}{\Delta t} & 0 \\ 0 & k_{12}\Delta t & 0 & -\frac{m}{\Delta t} & \left(\frac{2m}{\Delta t}+k\Delta t\right) & -\frac{m}{\Delta t} \\ 0 & 0 & k_{12}\Delta t & 0 & -\frac{m}{\Delta t} & \left(\frac{m}{\Delta t}+k\Delta t\right) \end{bmatrix} \tag{17}$$

on our graph. Thus, we borrow (loosely) from Wise [58] and suggest $\mathbf{K} \propto \partial_1 \cdot \partial_1^T$ since

$$\partial_1 \cdot \partial_1^T = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \tag{18}$$

produces precisely the same form as (17) and quantum theory is known to be "rooted in this harmonic paradigm" [62]. [In fact, these matrices will continue to have the same form as one increases the number of vertices in Fig. 3.] Now we construct a suitable candidate for $\mathbf{J}$, relate it to $\mathbf{K}$ and infer our SCC.

Recall that $\mathbf{J}$ has a component associated with each vertex so here it has components, J_n, $n = 1, 2, \ldots, 6$; J_n for $n = 1, 2, 3$ represents one source and J_n for $n = 4, 5, 6$ represents the second source. We propose $\mathbf{J} \propto \partial_1 \cdot \mathbf{e}$, where e_i are the

links of our graph, since

$$\partial_1 \cdot \mathbf{e} = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{bmatrix} = \begin{bmatrix} -e_1 - e_4 \\ e_1 - e_2 - e_3 \\ e_3 - e_7 \\ e_4 - e_5 \\ e_2 + e_5 - e_6 \\ e_6 + e_7 \end{bmatrix} \quad (19)$$

automatically makes $\mathbf{J}$ divergence-free, i.e., $\sum_i J_i = 0$, and relationally defined. Such a relationship on discrete spacetime lattices is not new. For example, Sorkin showed that charge conservation follows from gauge invariance for the electromagnetic field on a simplicial net [63].

With these definitions of $\mathbf{K}$ and $\mathbf{J}$ we have, *ipso facto*, $\mathbf{K} \cdot \mathbf{v} \propto \mathbf{J}$ as the basis of our SCC since

$$\partial_1 \cdot \partial_1^T \cdot \mathbf{v} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} -e_1 - e_4 \\ e_1 - e_2 - e_3 \\ e_3 - e_7 \\ e_4 - e_5 \\ e_2 + e_5 - e_6 \\ e_6 + e_7 \end{bmatrix} = \partial_1 \cdot \mathbf{e}$$

$$(20)$$

where we have used $e_1 = v_2 - v_1$ (etc.) to obtain the last column. You can see that the boundary of a boundary principle underwrites (20) by the definition of "boundary" and from the fact that the links are directed and connect one vertex to another, i.e., they do not start or end 'off the graph'. Likewise, this fact and our definition of $\mathbf{J}$ imply $\sum_i J_i = 0$, which is our graphical equivalent of a divergence-free, relationally defined source (every link leaving one vertex goes into another vertex). Thus, the SCC $\mathbf{K} \cdot \mathbf{v} \propto \mathbf{J}$ and divergence-free sources $\sum_i J_i = 0$ obtain tautologically via the boundary of a boundary principle. The SCC also guarantees that $\mathbf{J}$ resides in the row space of $\mathbf{K}$ so, as will be shown, we can avoid having to "throw away infinities" associated with gauge groups of infinite volume as in Faddeev-Popov gauge fixing. $\mathbf{K}$ has at least one eigenvector with zero eigenvalue which is responsible for gauge invariance, so *the self-consistent co-construction of space, time and divergence-free sources entails gauge invariance*.

Moving now to N dimensions, the Wick rotated version of (3) is (6) and the solution is (7). Using $\mathbf{J} = \alpha \partial_1 \cdot \mathbf{e}$ and $\mathbf{K} = \beta \partial_1 \cdot \partial_1^T$ ($\alpha, \beta \in \mathbb{R}$) with the SCC gives $\mathbf{K} \cdot \mathbf{v} = (\beta/\alpha)\mathbf{J}$, so that $\mathbf{v} = (\beta/\alpha)\mathbf{K}^{-1} \cdot \mathbf{J}$. However, $\mathbf{K}^{-1}$ does not exist because $\mathbf{K}$ has a nontrivial null space, therefore the row space of $\mathbf{K}$ is an $(N-1)$-dimensional subspace of the N-dimensional vector space.[5] The eigenvector with eigenvalue of zero, i.e., normal to this hyperplane, is $[1\ 1\ 1\ \ldots\ 1]^T$, which follows from the SCC as shown supra. Since $\mathbf{J}$ resides in the row space of $\mathbf{K}$ and, on our view, Z is a

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[5]This assumes the number of degenerate eigenvalues always equals the dimensionality of the subspace spanned by their eigenvectors.

functional of **K** and **J** which produces a partition function for the various $\mathbf{K}/2 + \mathbf{J}$ associated with different 4D experimental configurations, we restrict the path integral of (6) to the row space of **K**. Thus, our approach revises (7) to give (9).

Since this is linear, we do not expect to recover GR in this manner. Instead, we expect to make correspondence with GR via a modification to Regge calculus, a form of lattice gravity.

3 Recovering General Relativity: RBW Versus Hiley's Implicate Order

The modeling of "undivided wholeness" (monism) in each formalism leads to the same problem for both approaches when dealing with GR, i.e., how to relate/connect different M4 frames. This is simply to say the essence of gravity in GR is spacetime curvature, i.e., the relative acceleration of 'neighboring' geodesics, whereas the other forces are modeled via deviation from geodetic motion in a flat spacetime. Consider, for example, the phenomenon of gravitational lensing that produces an Einstein ring image of a distant quasar by an intervening galaxy. The explanation per GR is that empty spacetime around the worldtube of the intervening galaxy is curved so that null geodesics near its worldtube are deformed or 'bent' thereby 'lensing' the photons as they proceed from the quasar around the galaxy to Earth. We note that the principle explanatory mechanism, i.e., spacetime curvature, doesn't have anything to do with the stress-energy tensor of the quasar, or of the photons passing through that region of space, or of Earth. Yet, the monistic view doesn't allow for a separation of this sort—if we're relating the quasar, galaxy, photons, and Earth, then the stress-energy tensor for all these objects must be produced together with the geometry of spacetime from a single 'entity'.

For Hiley, this 'entity' will be a process-based algebra of the implicate order. Specifically, he speculates, a deformed Poisson algebra obtained via the hidden Heisenberg algebra gives the Moyal algebra for relating phase information for our electromagnetic interactions. If he proceeds with a current algebra approach (again, as inferred by his approach to Schrödinger, Pauli and Dirac particles), presumably, he will have to promote the spacetime metric to a field so that it will have its own particle and current. Then, he will have to produce commutation relations between the electromagnetic current and the gravitational current to describe the possible outcomes at interaction vertices. The problem is, of course, there are no spacetime locations for the interaction vertices, since one result of the calculation itself must be the spacetime geometry. Of course, if this algebra produces dual currents as with the Dirac particle, one is again left with the problem of figuring out which currents correspond to actual detector outcomes. But, suppose he takes the hint from his Dirac result and gives up on the idea of "Bohmian trajectories," as he has with the "Bohmian guidance equation," [37–39] and proceeds with a canonical quantization. Since his shadow manifolds are particular Lorentz frames rather than the full M4 for the Dirac equation, the logical counterpart to his approach (if it exists) for GR would be a particular foliation of the curved spacetime manifold. That is, a shadow manifold would be a particular path through all possible three geometries and matter fields in the solution space of $H = 0$.
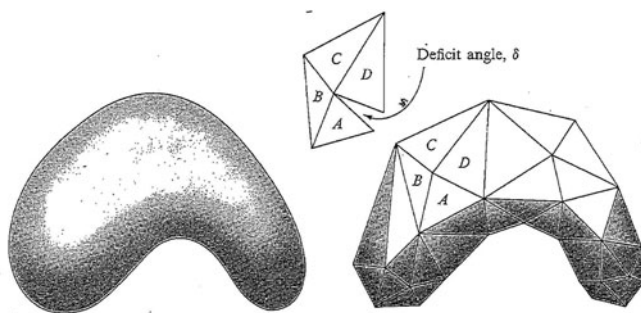
Figure 42.1.
A 2-geometry with continuously varying curvature can be approximated arbitrarily closely by a polyhedron built of triangles, provided only that the number of triangles is made sufficiently great and the size of each sufficiently small. The geometry in each triangle is Euclidean. The curvature of the surface shows up in the amount of deficit angle at each vertex (portion *ABCD* of polyhedron laid out above on a flat surface).

Fig. 5 Reproduced from Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. W.H. Freeman, San Francisco (1973), p. 1168. Permission pending

For RBW, the single 'entity' responsible for its monism is spacetimematter and we note immediately that for us the GR explanation of the Einstein ring in the above example must be corrected since there is no "empty spacetime." Thus, per RBW, GR is only an approximation to the 'correct' theory of gravity. Of course this is not new, the same can be said of Newtonian gravity given GR and Newtonian mechanics given special relativity. The questions are, what is the 'correct' theory of gravity and in what sense is it approximated by GR? Since our underlying approach is graphical, we start with the graphical version of GR, called Regge calculus, and propose modifications thereto.

In Regge calculus, the spacetime manifold is replaced by a lattice geometry where each cell is Minkowskian (flat). Typically, this lattice spacetime is viewed as an approximation to the continuous spacetime manifold, but the opposite could be true and that is what we will advocate. The lattice reproduces a curved manifold as the cells (typically 4D 'tetrahedra' called "simplices") become smaller (Fig. 5). Curvature is represented by "deficit angles" (Fig. 5) about any plane orthogonal to a "hinge" (triangular side to a tetrahedron, which is a side of a simplex). A hinge is two dimensions less than the lattice dimension, so in 2D a hinge is a zero-dimensional point (Fig. 5). The Hilbert action for a vacuum lattice is $I_R = \frac{1}{8\pi} \sum_{\sigma_i \in L} \varepsilon_i A_i$ where σ_i is a triangular hinge in the lattice L, A_i is the area of σ_i and ε_i is the deficit angle associated with σ_i. The counterpart to Einstein's equations is then obtained by demanding $\frac{\delta I_R}{\delta \ell_j^2} = 0$ where ℓ_j^2 is the squared length of the jth lattice edge, i.e., the metric. To obtain equations in the presence of matter-energy, one simply adds the matter-energy action I_M to I_R and carries out the variation as before to obtain $\frac{\delta I_R}{\delta \ell_j^2} = -\frac{\delta I_M}{\delta \ell_j^2}$. One finds the stress-energy tensor is associated with lattice edges, just as the metric, and Regge's equations are to be satisfied for any particular choice of the two tensors on the lattice. Thus, Regge's equations are, like Einstein's equations, a self-consistency criterion for the stress-energy tensor and metric.

It seems to us that the most glaring deviation from GR phenomena posed by directly connected sources per theory X would be found in the exchange of photons on cosmological scales. Therefore, using Regge calculus, we constructed a Regge differential equation for the time evolution of the scale factor $a(t)$ in the Einstein-de Sitter cosmology model (EdS) and proposed two modifications to the Regge calculus approach: (1) we allowed the graphical links on spatial hypersurfaces to be large, as when the interacting sources reside in different galaxies, and (2) we assumed luminosity distance D_L is related to graphical proper distance D_p by the equation $D_L = (1 + z)\sqrt{\overrightarrow{D_p} \cdot \overrightarrow{D_p}}$, where the inner product can differ from its usual trivial form [64]. There are two reasons we made this second assumption. First, in our view, space, time and sources are co-constructed, yet D_p is found without taking into account EM sources responsible for D_L. That is to say, in Regge EdS (as in EdS) we assume that pressureless dust dominates the stress-energy tensor and is exclusively responsible for the graphical notion of spatial distance D_p. However, even though the EM contribution to the stress-energy tensor is negligible, EM sources are being used to measure the spatial distance D_L. Second, in our view, there are no "photon paths being stretched by expanding space," so we cannot simply assume $D_L = (1 + z)D_p$ as in EdS. The specific form of $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ that we used to find the inner product for D_L was borrowed from linearized gravity in the harmonic gauge, i.e., $\partial^2 h_{\alpha\beta} = -16\pi G(T_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}T)$. That is, $D_L = (1 + z)\sqrt{1 + h_{11}}D_p$ and we use $\mathbf{K} \cdot \mathbf{Q}_0 = \mathbf{J}$ to find h_{11}. We emphasize that $h_{\alpha\beta}$ here corrects the graphical inner product $\eta_{\alpha\beta}$ in the inter-nodal region between the worldlines of photon emitter and receiver, where $\eta_{\alpha\beta}$ is obtained via a matter-only stress-energy tensor. Since the EM sources are negligible in the matter-dominated solution and we're only considering a classical deviation from a classical background, we have $\partial^2 h_{\alpha\beta} = 0$ to be solved for h_{11}. Obviously, $h_{11} = 0$ is the solution that gives the trivial relationship, but allowing h_{11} to be a function of D_p allows for the possibility that D_L and D_p are not trivially related. We have $h_{11} = AD_p + B$ where A and B are constants and, if the inner product is to reduce to $\eta_{\alpha\beta}$ for small D_p, we have $B = 0$. Presumably, A should follow from the corresponding theory of quantum gravity, so an experimental determination of its value provides a guide to quantum gravity per our view of classical gravity. As we will show, our best fit to the Union2 Compilation data gives $A^{-1} = 8.38$ Gcy, so the correction to η_{11} is negligible except at cosmological distances, as expected.

The modified Regge calculus model (MORC), EdS and the concordance model ΛCDM (EdS plus a cosmological constant to account for dark energy) were compared using the data from the Union2 Compilation, i.e., distance moduli and redshifts for type Ia supernovae [65] (see Figs. 6 and 7). We found that a best fit line through $\log(\frac{D_L}{\text{Gpc}})$ versus $\log z$ gives a correlation of 0.9955 and a sum of squares error (SSE) of 1.95. By comparison, the best fit ΛCDM gives SSE $= 1.79$ using $H_o = 69.2$ km/s/Mpc, $\Omega_M = 0.29$ and $\Omega_\Lambda = 0.71$. The parameters for ΛCDM yielding the most robust fit to "the Wilkinson Microwave Anisotropy Probe data with the latest distance measurements from the Baryon Acoustic Oscillations in the distribution of galaxies and the Hubble constant measurement [66]" are $H_o = 70.3$ km/s/Mpc, $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$, which are consistent with the parameters we find for its Union2 Compilation fit. The best fit EdS gives SSE $= 2.68$ using $H_o = $

Fig. 6 Plot of transformed Union2 data along with the best fits for linear regression (*thin black*), EdS (*dashed*), ΛCDM (*gray*), and MORC (*dotted*)
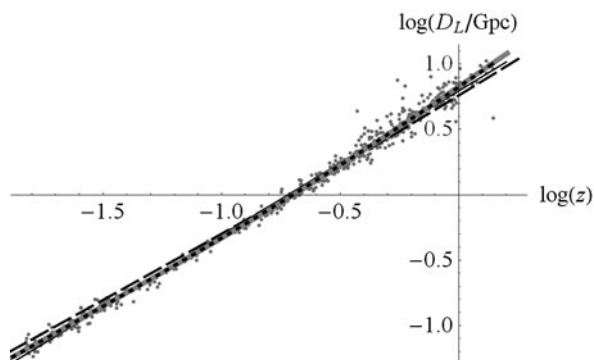
Fig. 7 Plot of Union2 data along with the best fits for EdS (*dashed*), ΛCDM (*gray*), and MORC (*dotted*). The MORC curve is terminated at $z = 1.4$ in this figure so that the ΛCDM curve is visible underneath
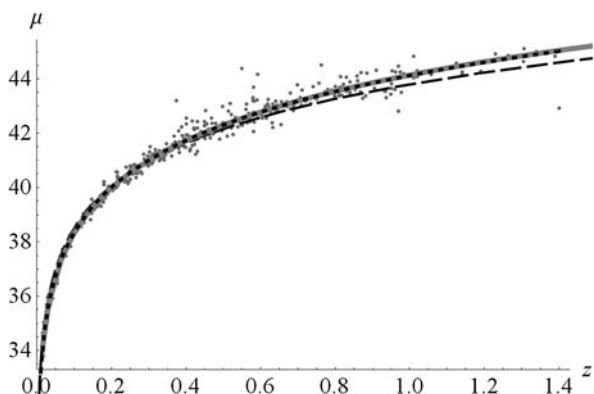
60.9 km/s/Mpc. The best fit MORC gives SSE $= 1.77$ and $H_o = 73.9$ km/s/Mpc using $R = A^{-1} = 8.38$ Gcy and $m = 1.71 \times 10^{52}$ kg, where R is the coordinate distance between nodes, A^{-1} is the scaling factor from our non-trivial inner product explained above, and m is the mass associated with nodes.[6] A current (2011) "best estimate" for the Hubble constant is $H_o = (73.8 \pm 2.4)$ km/s/Mpc [67]. Thus, MORC improves EdS as much as ΛCDM in accounting for distance moduli and redshifts for type Ia supernovae even though the MORC universe contains no dark energy is therefore always decelerating.

This is but one test of the RBW approach and MORC must pass more stringent tests in the context of the Schwarzschild solution where GR is well confirmed. However, MORC's empirical success in dealing with dark energy gives us reason to believe this formal approach to classical gravity may provide creative new techniques for solving other long-standing problems, e.g., quantum gravity, unification, and dark matter. In particular, if MORC passes empirical muster in the context of the Schwarzschild solution, then information such as A^{-1} might provide guidance to a theory of quantum gravity underlying a graphical classical theory of gravity.

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[6]Strictly speaking, the stress-energy tensor is associated with graphical links, not nodes. Our association of mass with nodes is merely conceptual.

4 The Problems of Time: RBW Versus the Implicate Order on Being and Becoming

4.1 The Implicate Order

It is obvious that a process conception of fundamental reality does not sit well with blockworld or frozen time. In the case of blockworld there is no unique 'now' successively coming into existence. There are an indenumerably infinite number of time-like foliations of M4, each representing a unique global 'now' at various values of its foliating time, and a particular spatial hypersurface in foliation A (a 'now' for observer A) contains events on many different spatial hypersurfaces in foliation B (different 'nows' for observer B). That events which are simultaneous for observer A are not simultaneous for observer B is called the "relativity of simultaneity" and negates an objective passage of time. That is to say, there is no objective (frame independent) distinction in spacetime between past, present and future events respectively and therefore no objective distinction to be had about the occurrence or non-occurrence of events. In the words of Costa de Beauregard [68]:

> This is why first Minkowski, then Einstein, Weyl, Fantappiè, Feynman, and many others have imagined space-time and its material contents as spread out in four dimensions. For those authors, of whom I am one ... relativity is a theory in which everything is "written" and where change is only relative to the perceptual mode of living beings.

And we have seen that the canonical or gauge interpretation of GR leads to an even "blockier" world than SR! As Earman puts it [69]:

> Taken at face value, the gauge interpretation of GTR implies a truly frozen universe: not just the 'block universe' that philosophers endlessly carp about— that is, a universe stripped of A-series change or shifting 'nowness'—but a universe stripped of its B-Series change in that no genuine physical magnitude (= gauge invariant quantity) changes its value with time.

As for the problem of frozen time in canonical QG, as we said, the dynamics of the theory are given by a Hamiltonian operator $\hat{H}$, which is defined on a space of spin network states via the equation $\hat{H}|\Psi\rangle = 0$, i.e., the Wheeler-DeWitt equation mentioned earlier. It is hard to see how to avoid the problem of frozen time in canonical QG because, unlike the standard Schrödinger equation $\hat{H}|\Psi\rangle = i\hbar\frac{\partial|\Psi\rangle}{\partial t}$, the RHS of the Wheeler-DeWitt equation disappears. Because time is part of the physical system being quantized, there is no external time with respect to which the dynamics could unfold, only the analogous gauge symmetries are there.

Therefore, in order to preserve his process model of reality, at the end of the day Hiley must end up with a fundamental physical theory that avoids the blockworld of relativity and the frozen time of canonical QG. We can only speculate as to exactly how Hiley will address these concerns or even exactly how his program will recover GR, therefore the reader should consider our suggestions tentative. Let's discuss SR first and extrapolate from there.

Any argument from SR to blockworld requires, as a premise, realism about the geometric properties of M4. As we indicated earlier, we think Hiley might be in a position to reject such realism because in his scheme each shadow manifold constitutes a particular Lorentz frame. Every Lorentz observer will construct his own space and time. These space-times can exist together, but we cannot ascribe a sharply defined 'time' as to when they all exist together. Therefore every frame is its own coordinate origin of its own explicate manifold. Think of the implicate order as the head of an octopus and the various explicate shadow manifolds (e.g., individual perspectives or proper times) as the many tentacles produced from the implicate order by the holomovement. Every event is described by an infinity of times and spatial locations even though there is only one event that all Lorentz observers are observing, as related formally by the Lorentz group. The shadow manifolds are not connected directly and thus there is no M4 as conceived by Minkowski-there isn't one spacetime. As for the problem of time in canonical GR and in canonical QG, again, assuming he gives up on Bohmian trajectories and guide waves and uses canonical quantization per his yet-to-be-determined process algebra for gravity and all other forces, then his shadow manifolds correspond to particular paths through all possible three geometries and matter fields in the solution space of $H = 0$. Thus, Hiley avoids "frozen time" in GR and QG exactly like he avoids it in SR—by giving up on the idea of a unique explicate order *a la* M4, leaving the unification of perspective to the implicate order as dictated by the holomovement. Whether or not such a view is Hiley's considered view and whether or not it is any better off than solipsism, we do not know. Hiley is clear however that blockworld defined as the reality of all events past, present and future is inconsistent with his process ontology. This means he either rejects realism about M4 at its root or provides a physically and formally acceptable preferred foliation in addition to the structure of M4.

Given that Hiley rejects blockworld it would be reasonable to assume that he embraces some form of presentism (only the present is real). However in his theory of moments [70] he clearly rejects presentism. According to Hiley's theory of moments, the holomovement gives rise to "moments/durons" (which involves information from the past and the future). Of moments he goes on to say that: "For a process with a given energy cannot be described as unfolding at an instant except in some approximation" [71]. As we understand it, the idea is that the holomovement can explicate either a small region or a large region of spacetime (to include the future) 'at once', though never the entire universe. The extent of the explicate domain (how much of the future exists) depends on the properties of the holomovement in each particular case and the process is apparently stochastic. In Hiley's model therefore, just as the past can effect what unfolds in the future, so the future can influence what unfolds in the present and what unfolded in the past. Hiley is clear that what happens in the future cannot be made to rewrite the past, but that the future possibilities can influence the unfolding of the present. What is less clear is whether these moments pass in and out of existence or always stay in existence once explicated. All this suggests that each individual shadow manifold is constantly changing in its own time (evolving 'now') such that the past is consistent with the present and the future is understood probabilistically. Again, the solipsistic view of individual shadow manifolds connected via the implicate order per the holomovement avoids the blockworld implication of M4.

One could imagine other hybrid models of blockworld and presentism (or at least becoming) such as entire blockworld universes winking discretely in and out existence, each one different in some way from the last. How to formalize models such as these, whether in an algebraic program or some other, is unclear to us. What is clear to us at the end of the day, merely advocating for fundamental physics based on process isn't enough to secure every feature of dynamism. Whether or not quantum theory and relativity can be unified in such a way as to uphold all of dynamism is a formal question that has yet to be resolved.

4.2 RBW

Of course we happily accept the implication of relativity theory that it is a block universe and we are not bothered by the problem of frozen time in canonical QG because we reject dynamism at its foundation. For those wedded to dynamism these results are puzzling embarrassments that require some sort of compatibilist response or a completely new process-based ontology and formalism. In RBW we start at bottom with an adynamical global constraint, a self-consistency criterion (SCC) that allows us to construct discrete spacetimematter graphs from which all the other effective theories and their concomitant phenomena emerge. According to RBW, what quantum theory and relativity theory are both trying to tell us is that every facet of dynamism is false. If we succeed in our program of unification, we will have shown that nothing in physics itself demands dynamism, rather it was just a historical contingency based in the fact that all physics must start with experience. Perhaps RBW offers a fourth possibility regarding the nature of time, i.e., time as part of a fundamental (pregeometric) regime wherein the notions of space, time and matter are co-defined and co-determining. Technically, time, space and matter as stand-alone concepts are not fundamental, emergent or illusions in RBW. We note that it is only from a God's eye Point of view (the view from nowhere and nowhen) that time and change are an illusion and in a fundamentally relational model such as ours there are no perspectives "external to the universe." The conceptual foundation of our dynamical reality isn't a so-called "initial singularity," but the adynamical SCC upon which all dynamic theories reside. The SCC characterizing spacetimematter at the bottom of RBW is not a dynamical law or initial condition, but it is responsible for the discrete action. Therefore, if higher-level physical theories are truly recovered from the discrete action, then there is nothing left to explain at bottom, regardless what phenomena one counts as initial/boundary conditions versus laws. The point of all this is that in RBW there will be no quantum cosmology as is currently conceived. We also note that the universe comes with many physically significant modes of temporal passage and change such as proper time, cosmic time, etc. Certainly these constitute objective notions of becoming (objectively dynamical flow) even if they are mere patterns in a block universe. Therefore, RBW does not negate change and becoming, it merely internalizes and relativizes them.

Of course, all this falls short of getting every facet of time as experienced into fundamental physics. There is no objectively distinguished present moment, and there is no objectively dynamical becoming in the sense of bringing events into existence that never existed before from a God's eye point of view. However, perhaps the standard wisdom that time as experienced is either a physical feature of reality or merely

a psychological feature of conscious beings is a false dichotomy. Perhaps what all this suggests is that conscious temporal experience is fundamental as well, so instead of spacetimematter at bottom we have the super-monistic spacetimematterexperience at bottom. This is shear speculation of course, it would require working out a new formal model and much else conceptually. We can say however that the alternatives are not very appetizing if we take the frozen block universe seriously. The image of consciousness crawling along the worldtube of individuals illuminating the present and moving it toward the future is an unhelpful and non-explanatory kind of dualism which simply exempts conscious experience from the rules of the block universe [72]. The other alternative, that conscious experience emerges from or is realized in neuro-dynamical activity, is problematic in a block universe in which everything, past, present and future is just there 'at once' (including conscious experiences throughout the block) and brains are just worldtubes like everything else. One might find correlations between brain states and the experience of the objective specialness of the 'now' and the experience of objectively dynamical becoming, but it cannot be said that brain dynamics produce or bring into being conscious states (themselves worldtubes). In such a universe brain processes are not metaphysically or causally more fundamental than conscious processes. Again, the idea of spacetimematterexperience is half-baked, but if we take it seriously, perhaps it moves RBW closer to Hiley's process conception of reality since process (objectively distinguished present and objectively dynamical) is the nature of ordinary conscious experience and the experience of time partially motivates the process model.

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