

Supplemental Appendix for Survival , Attrition and Biased Decision-Making

i) Note that $\partial B_n / \partial n$ is clearly positive since:

$$\frac{\partial B_n}{\partial n} = \frac{\sigma_\varepsilon \sigma_\mu^2 (\bar{s} - g(\bar{s}))}{(\sigma_\varepsilon^2 + n\sigma_\mu^2) \int_{(\beta-\hat{p})/\sigma_\varepsilon}^{\infty} e^{-1/2t^2} dt}$$
 with all parameters larger than zero and $E(\bar{s} - g(\bar{s}))$ free of

n as well.

ii) In the case of $\partial B_n / \partial \sigma_\varepsilon$, there are no clear-cut signs of the derivative; parameters produce both positive and negative results. Nonetheless, some bounds could be established, for example in order for $\partial B_n / \partial \sigma_\varepsilon > 0$ there is a necessary and sufficient condition

Proof:

Note that after taking expectations:

$$\begin{aligned} \frac{\partial B_n}{\partial \sigma_\varepsilon} &= \frac{2e^{-\frac{(\beta-\mu)^2}{2\sigma_\varepsilon^2}} \frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2} n(\beta-\mu)\sigma_\mu^2}{\pi\sigma_\varepsilon^2(n\sigma_\mu^2+\sigma_\varepsilon^2)(1+\text{Erf}\left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon}\right])^2} - \frac{2ne^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} \sqrt{\frac{2}{\pi}}\sigma_\mu^2\sigma_\varepsilon^2}{(n\sigma_\mu^2+\sigma_\varepsilon^2)^2(1+\text{Erf}\left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon}\right])} + \frac{e^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} \sqrt{\frac{2}{\pi}}n\sigma_\mu^2}{(n\sigma_\mu^2+\sigma_\varepsilon^2)(1+\text{Erf}\left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon}\right])} + \\ &\frac{e^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} \sqrt{\frac{2}{\pi}}(\mu-\beta)^2n\sigma_\mu^2}{\sigma_\varepsilon^2(n\sigma_\mu^2+\sigma_\varepsilon^2)(1+\text{Erf}\left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon}\right])} \quad (A) \end{aligned}$$

Now to see how can $\partial B_n / \partial \sigma_\varepsilon > 0$ I re-arrange and simplify (A) equation to compare first

two terms to the second two, the resulting necessary and sufficient condition is then given by:

$$\frac{\partial B_n}{\partial \sigma_\varepsilon} > 0 \quad \text{iff} \quad \frac{2e^{-\frac{(\beta-\hat{p})^2}{2\sigma_\varepsilon^2}} (\hat{p}-\beta)}{\sqrt{2\pi}(1-\text{Erf}\left[\frac{\beta-\hat{p}}{\sqrt{2}\sigma_\varepsilon}\right])} + \frac{2\sigma_\varepsilon^4}{n\sigma_\mu^2+\sigma_\varepsilon^2} < \sigma_\varepsilon^2 + (\hat{p}-\beta)^2$$

From which it clearly follows that assuming $\sigma_\varepsilon^2 > \frac{2\sigma_\mu^4}{n\sigma_\mu^2 + \sigma_\varepsilon^2}$, $(\hat{p} - \beta)^2 > \frac{\frac{(\beta - \hat{p})^2}{2\sigma_\varepsilon^2} (\hat{p} - \beta)}{\sqrt{2\pi}(1 - \text{Erf}(\frac{\beta - \hat{p}}{\sqrt{2}\sigma_\varepsilon})}$ are

a sufficient and necessary condition for above inequality to hold ■

There is also a range of parameters for which $\partial B_n / \partial \sigma_\varepsilon < 0$. It is clear that reversing the inequality signs produces the two necessary and sufficient conditions for that range of parameters.

iii) $\partial B_n / \partial \sigma_\mu > 0$ for all values of parameters, this follows from the fact that differentiating Bias

with respect to prior variance yields:

$$\frac{\partial B_n}{\partial \sigma_\mu} = \frac{2 e^{-\frac{(\mu - \beta)^2}{2\sigma_\varepsilon^2}} n^2 \sqrt{\frac{2}{\pi}} \sigma_\mu \sigma_\varepsilon}{(n\sigma_\mu^2 + \sigma_\varepsilon^2)(1 + \text{Erf}\left[\frac{\mu - \beta}{\sqrt{2}\sigma_\varepsilon}\right])} - \frac{2 e^{-\frac{(\mu - \beta)^2}{2\sigma_\varepsilon^2}} n \sqrt{\frac{2}{\pi}} \sigma_\mu^3 \sigma_\varepsilon}{(n\sigma_\mu^2 + \sigma_\varepsilon^2)^2 (1 + \text{Erf}\left[\frac{\mu - \beta}{\sqrt{2}\sigma_\varepsilon}\right])}$$

which can be easily simplified to:

$$\frac{\partial B_n}{\partial \sigma_\mu} = \frac{2 e^{-\frac{(\mu - \beta)^2}{2\sigma_\varepsilon^2}} \sqrt{\frac{2}{\pi}} \sigma_\varepsilon^3 n \sigma_\mu}{(n\sigma_\mu^2 + \sigma_\varepsilon^2)^2 (1 + \text{Erf}\left[\frac{\mu - \beta}{\sqrt{2}\sigma_\varepsilon}\right])}$$

It is immediately clear that $\partial B_n / \partial \sigma_\mu > 0$.

iv) $\partial B_n / \partial \beta$ could also take on both positive and negative values depending on parameters since

differentiating with respect to the truncation point yields:

$$\partial B_n / \partial \beta = \frac{2e^{-\frac{(\beta-\mu)^2}{2\sigma_\varepsilon^2}} \frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2} n\sigma_\mu^2}{\pi(n\sigma_\mu^2 + \sigma_\varepsilon^2) \left(1 + \operatorname{Erf} \left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon} \right]\right)^2} - \frac{e^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} \sqrt{\frac{2}{\pi}} (\mu-\beta) n\sigma_\mu^2}{\sigma_\varepsilon (n\sigma_\mu^2 + \sigma_\varepsilon^2) \left(1 + \operatorname{Erf} \left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon} \right]\right)} \quad (\text{B})$$

where in case of severe truncation ($\beta > \mu$) it is clear that $\partial B_n / \partial \beta > 0$. In the case of milder truncation ($\beta < \mu$) this result is more difficult to obtain. More specifically an additional condition needs to be imposed.

Rearranging (B), while assuming $\beta < \mu$, it becomes clear that

$$\partial B_n / \partial \beta > 0 \quad \text{iff} \quad 2\sigma_\varepsilon e^{-\frac{(\mu-\beta)^2}{2\sigma_\varepsilon^2}} > \sqrt{2\pi} (\hat{p} - \beta) \left(1 + \operatorname{Erf} \left[\frac{\mu-\beta}{\sqrt{2}\sigma_\varepsilon} \right]\right)$$

Dynamics

Note that while Equation 13 has no impact on bias we need to make sure solution exists and that is finite.

$$\text{solution to } \sigma_{t+1} = \frac{\sigma_\varepsilon^2 \sigma_{\mu(t)}^2}{\sigma_{\mu(t)}^2 + n\sigma_\varepsilon^2} \quad \text{is} \quad \sigma_{\mu(t)} = \frac{(n-1)n\sigma_\varepsilon^2}{n^{t+1} - n^t \sigma_\varepsilon^2 + n^{t+1} \sigma_\varepsilon^2 - n}$$

Assuming that initial condition constant is equal to 1. Clearly above solution converges to 0.